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Noise cleaning in binary-valued digital pictures using "propagation" processes

Azriel Rosenfeld*, Chan M. Park*, and James P. Strong**

Abstract

Methods of cleaning noise in a binary-valued digital picture by using "propagation" processes are described, with emphasis on the possibility of preserving elongated parts of objects while deleting small non-elongated parts.

The support of the U. S. Atomic Energy Commission, under Contract AT-(40-1)-3662 with the Computer Science Center of the University of Maryland, is gratefully acknowledged.

* Computer Science Center, University of Maryland
** Goddard Space Flight Center, National Aeronautics and Space Administration

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1. Introduction

One of the earliest papers on picture processing by computer [1] introduced a simple method of "cleaning up" noise in a binary-valued digital picture : 1's are changed to 0's if they have more than a certain number of 0's as neighbors, and vice versa. Since that time, many other methods of noise cleaning have been developed, including refinements of the neighbor-counting method [e.g., 2-3] and methods based on selective spatial frequency filtering [e.g., 4-6]. In addition, many investigators have developed noise cleaning techniques appropriate to particular classes of pictures, and have used them as part of the "preprocessing" for specific pictorial pattern recognition tasks.

The purpose of the present report is to describe an approach to noise cleaning based on applying "propagation" (or "distance"-computing) operations to the given picture. In particular, it is shown how this approach can be used to clean noise while preserving thin, elongated "signals".

LIST OF FIGURES

1. Introduction

One of the earliest papers on picture processing by computer [1] introduced a simple method of "cleaning up" noise in a binary-valued digital picture : 1's are changed to 0's if they have more than a certain number of 0's as neighbors, and vice versa. Since that time, many other methods of noise cleaning have been developed, including refinements of the neighbor-counting method [e.g., 2-3] and methods based on selective spatial frequency filtering [e.g., 4-6]. In addition, many investigators have developed noise cleaning techniques appropriate to particular classes of pictures, and have used them as part of the "preprocessing" for specific pictorial pattern recognition tasks.

The purpose of the present report is to describe an approach to noise cleaning based on applying "propagation" (or "distance"-computing) operations to the given picture. In particular, it is shown how this approach can be used to clean noise while preserving thin, elongated "signals".

2. Neighbor-counting methods and modifications

To illustrate the limitations of the classical approach to noise cleaning described in [1], consider the binary-valued digital picture shown in Figure 1. Here it might be reasonable to regard the circles (which look like ellipses because the vertical and horizontal spacing on the printer are unequal) as "signal", and the dots as "noise"; the streaks might be regarded as signal for some purposes, as noise for others. In Figure 1a, the probability that an element is a noise dot, or the center of a streak, is 0.025; in Figure 1b, 0.05; in Figure 1c, 0.10; and in Figure 1d, 0.20.

Let $D_{ij}^{(r)}$ be the set of elements at distance $< r + \frac{1}{2}$ from (i,j); let n_r be the number of elements in this set. Let $F_{r,\lambda}$ (where $0 \le \lambda \le 1$) be the operation which takes the binary-valued digital picture (a_{ij}) into the binary-valued digital picture (b_{ij}) defined as follows:

$$b_{ij} = 0 \text{ if the number of 1's in } D_{ij}^{(r)} \text{ is } \le \lambda\, n_r$$
$$= 1 \text{ if this number is } \ge (1-\lambda)n_r$$
$$= a_{ij} \text{ otherwise}$$

In other words, $F_{r,\lambda}$ leaves (a_{ij}) unchanged except (possibly) for elements having very many 0's or very many 1's as neighbors; such elements themselves become 0's or 1's, respectively.

Figures 2-8 show the results of applying $F_{r,\lambda}$ to Figure 1 for the following values of r and λ:

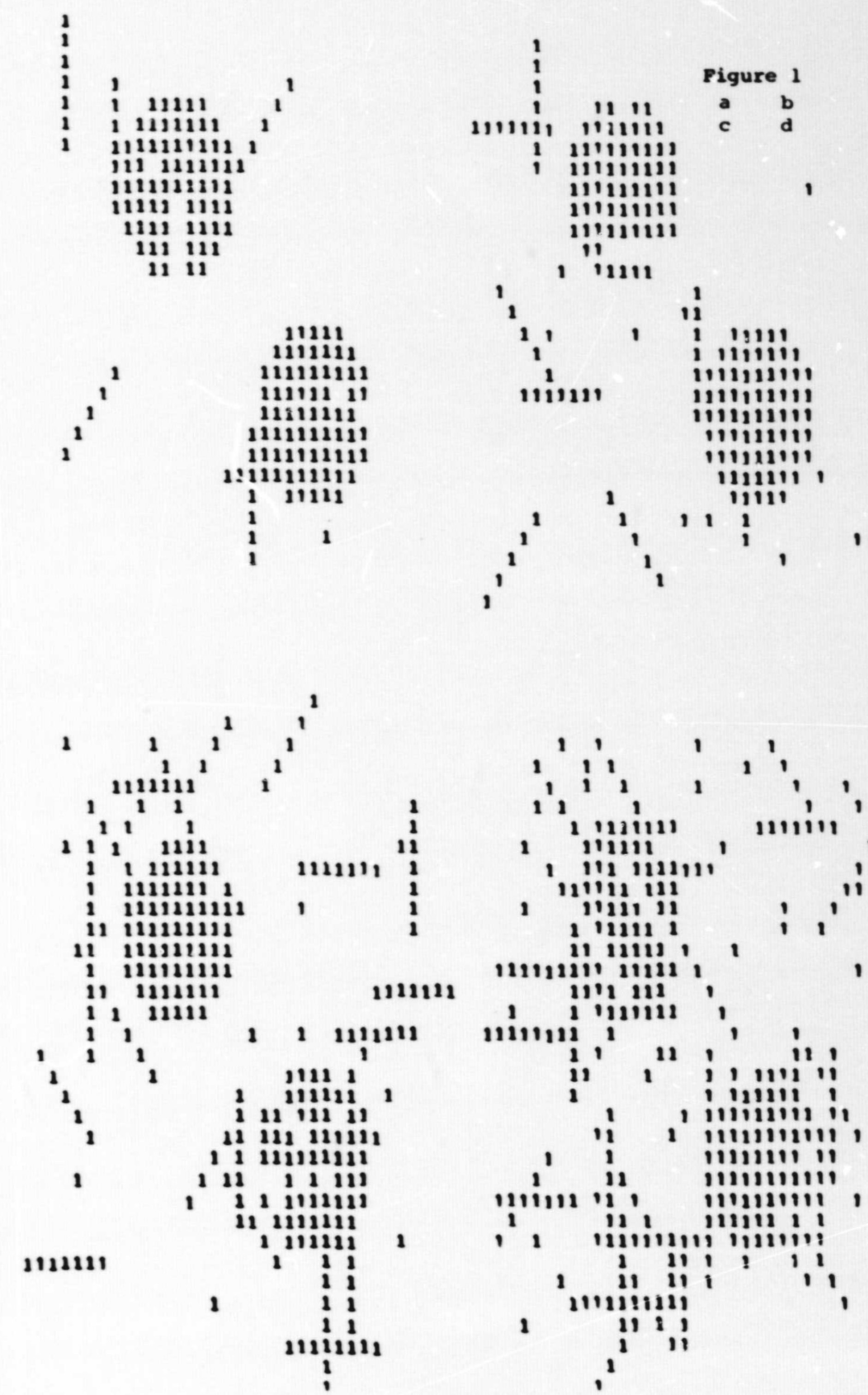


Figure 1
a b
c d

Figure	r	λ
2	1	0.2
3	1	0.3
4	1	0.4
5	2	0.1
6	2	0.2
7	2	0.3
8	2	0.4

Note that for r = 1, the set $D_{ij}^{(r)}$ is a 3-by-3 "square" consisting of (i,j) and its eight neighbors; while for r = 2, $D_{ij}^{(r)}$ is an "octagon" having 21 elements.

It is seen from these results that except for the lowest values of λ, these operations treat the streaks as noise; however, only for high λ are they eliminated entirely, while for lower λ's they shrink but do not completely disappear. (Note that for r = 2, diagonal streaks are treated more harshly than horizontal and vertical streaks, since the set $D_{ij}^{(r)}$ is five elements high and wide, but only three elements across in the diagonal directions.) Evidently, neighbor-counting methods cannot be effective for high levels of "dot" noise without also treating streaks as noise.

A further difficulty with these operations, as seen in Figures 2-8, is that noise which is close to the edge of the "signal" is usually not eliminated, since when (i,j) is such a noise element, $D_{ij}^{(r)}$ contains many "signal" elements. (This problem becomes increasingly troublesome as r increases.) To eliminate this objection, one can modify the neighbor-counting method along the following lines (compare [7]):

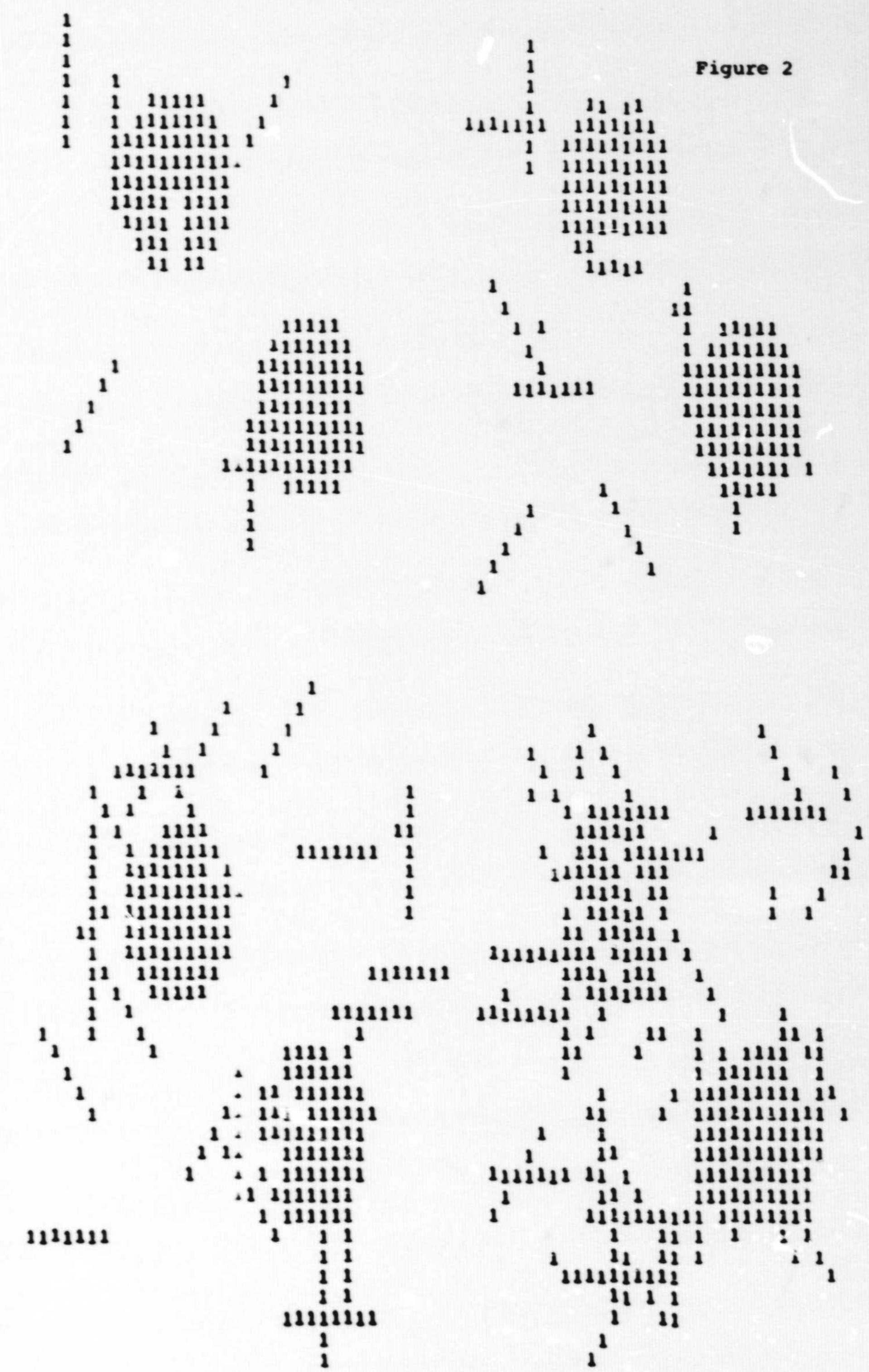


Figure 2

Figure 3

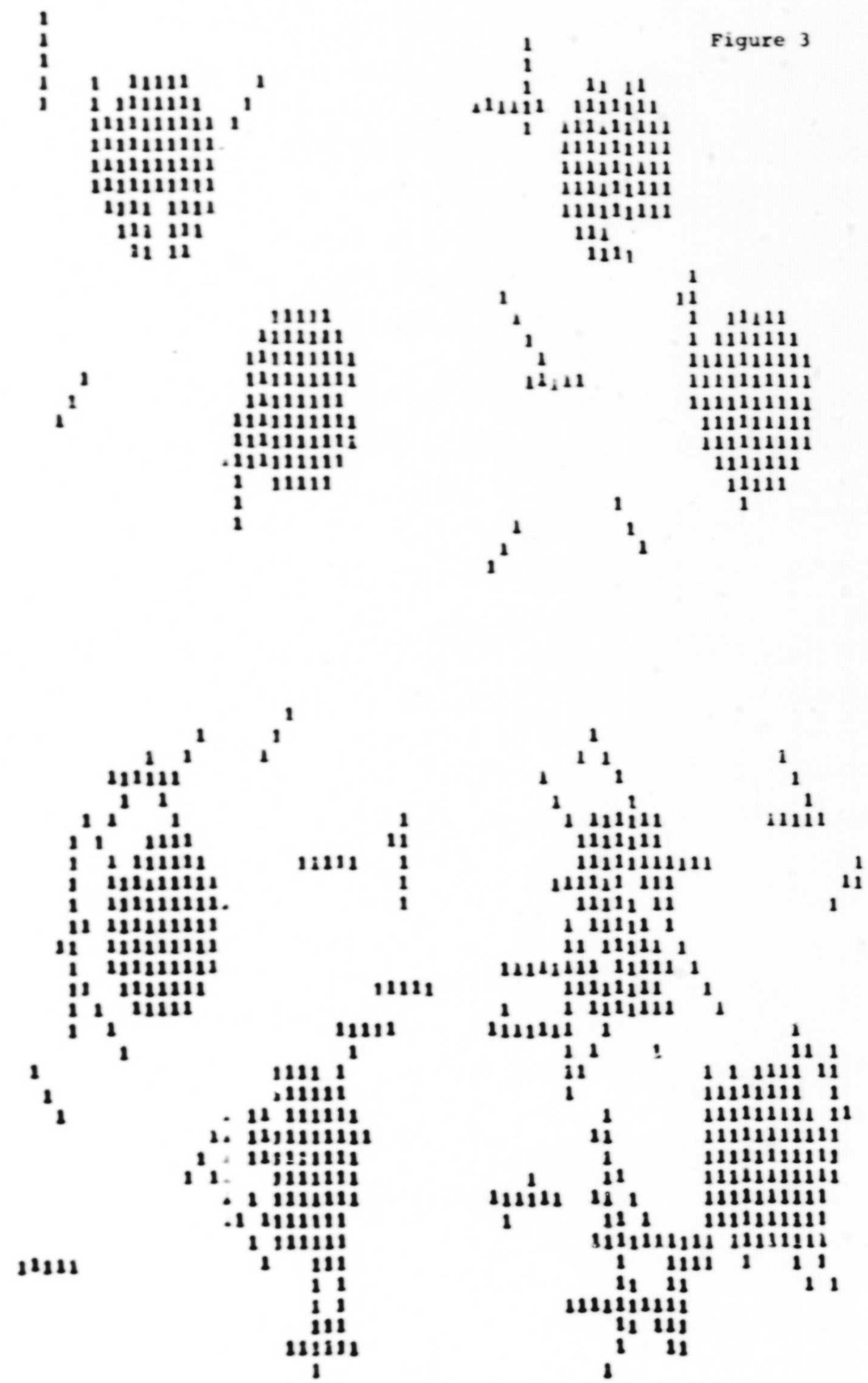
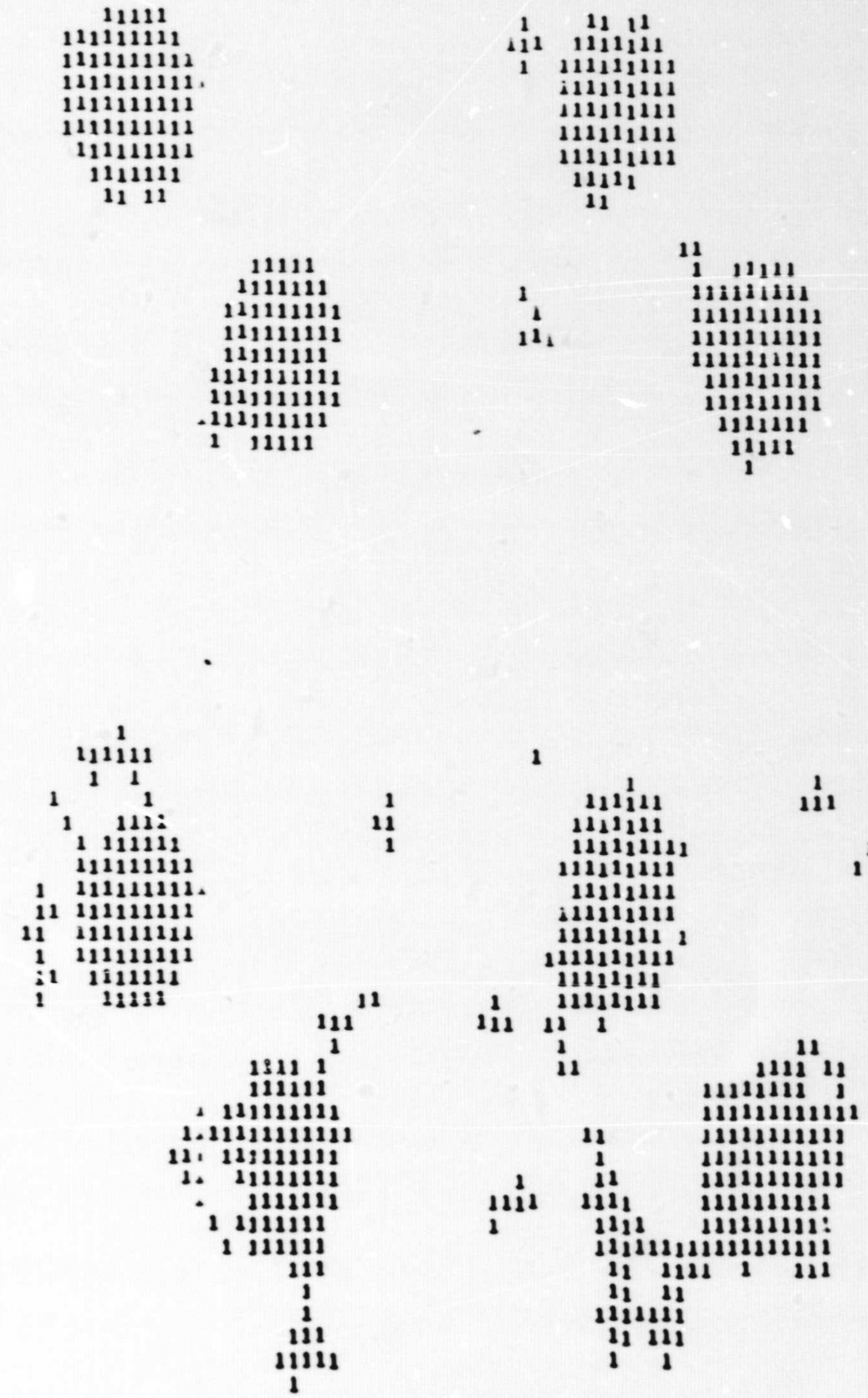


Figure 4



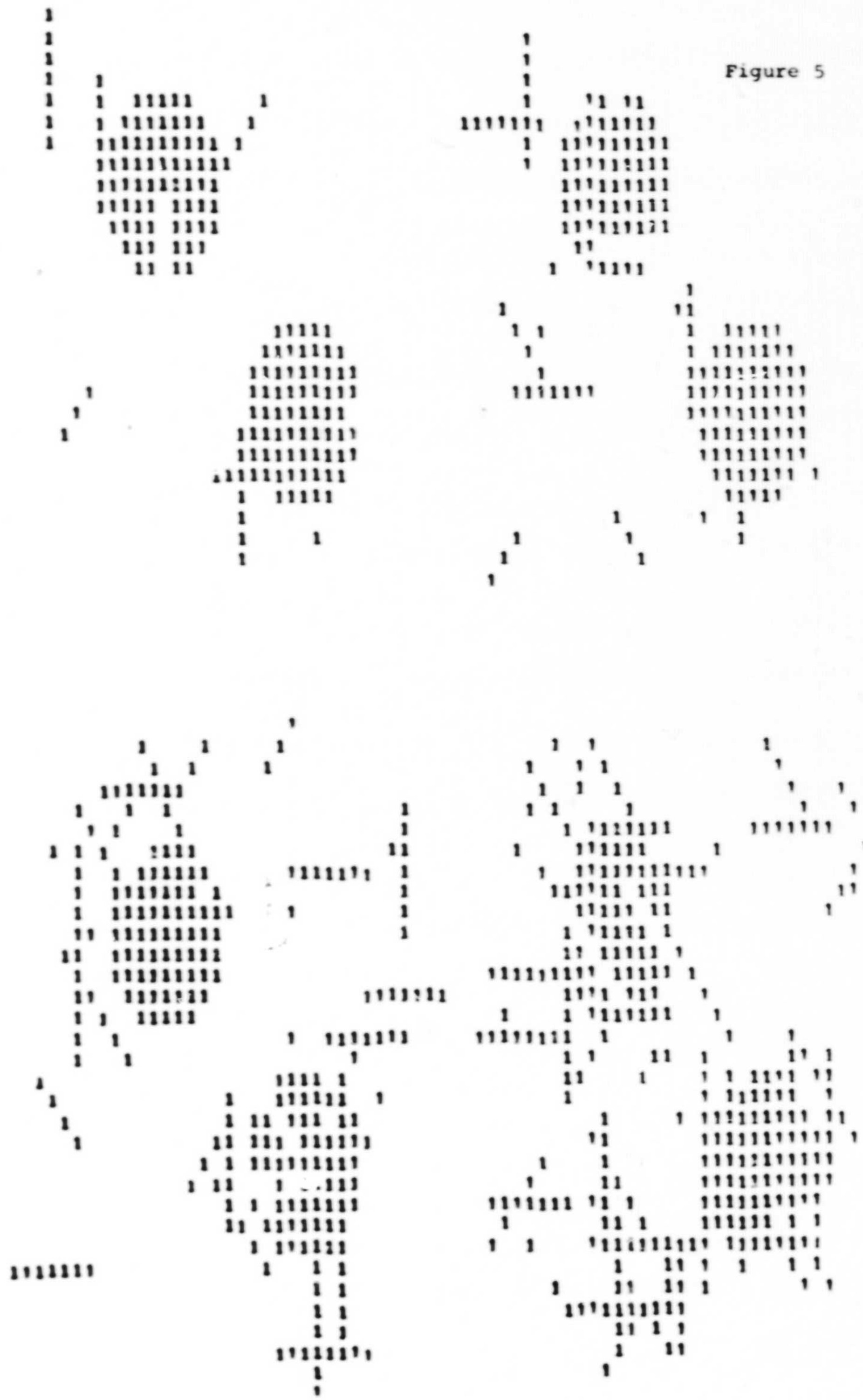


Figure 5

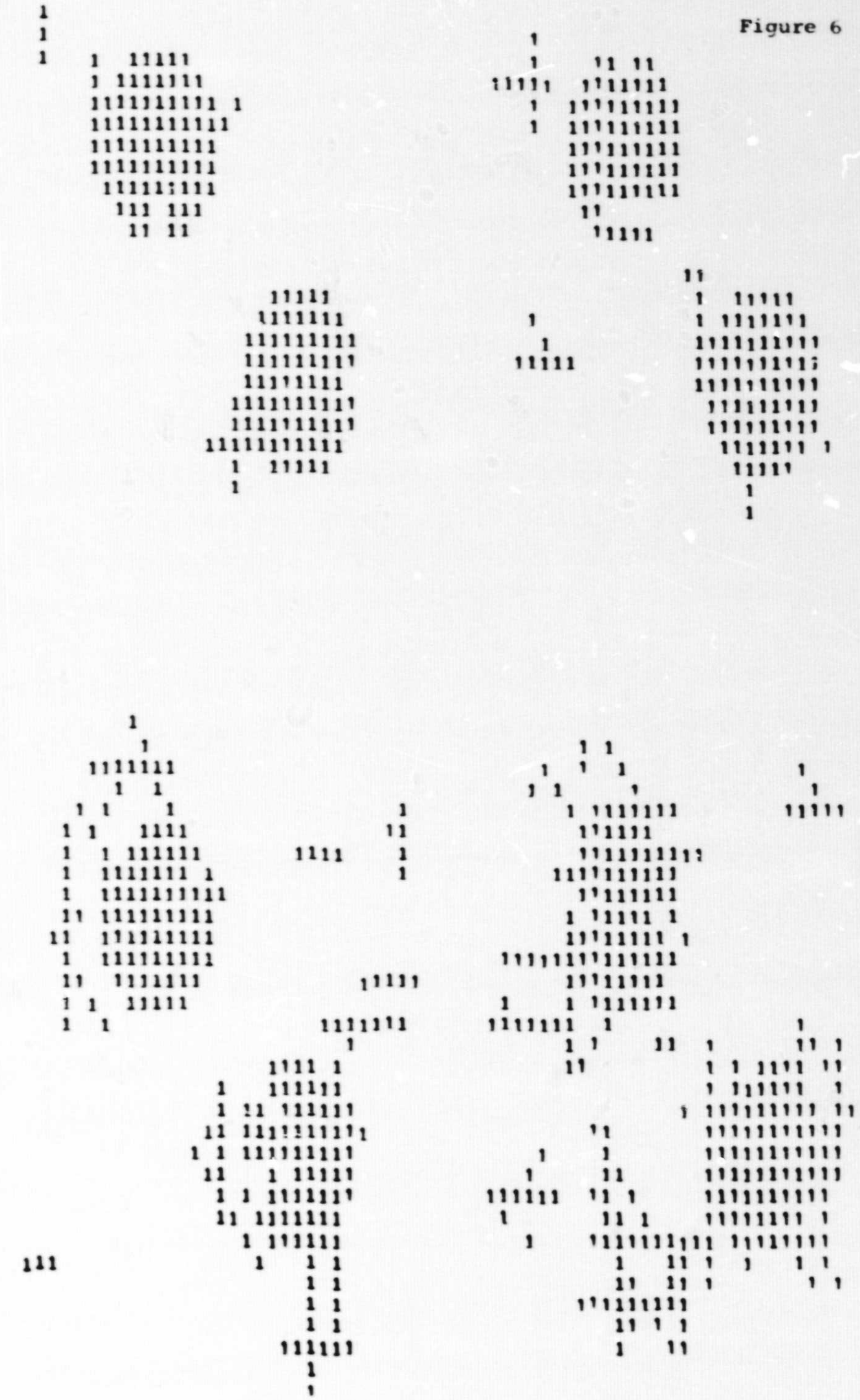


Figure 6

Figure 7

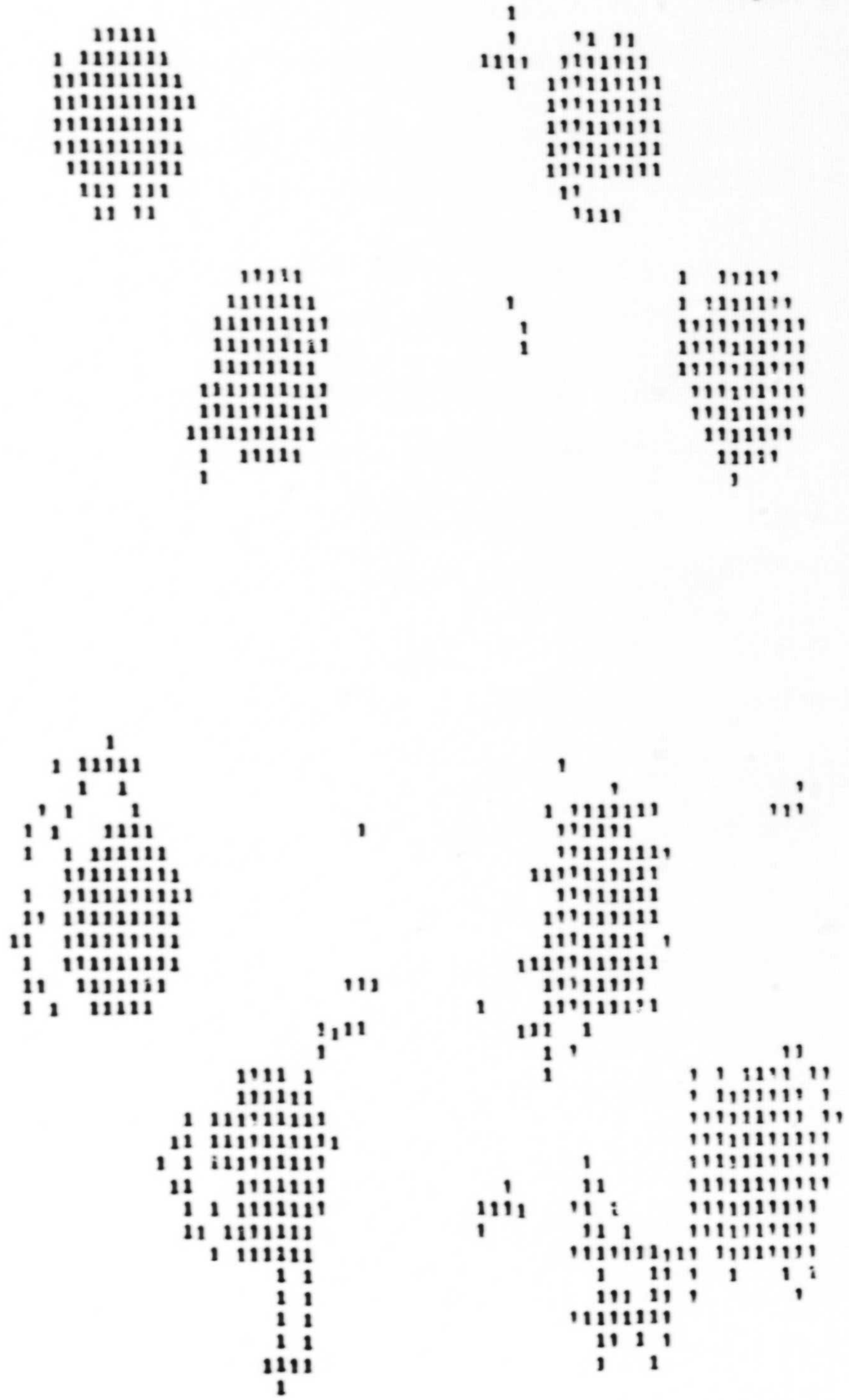
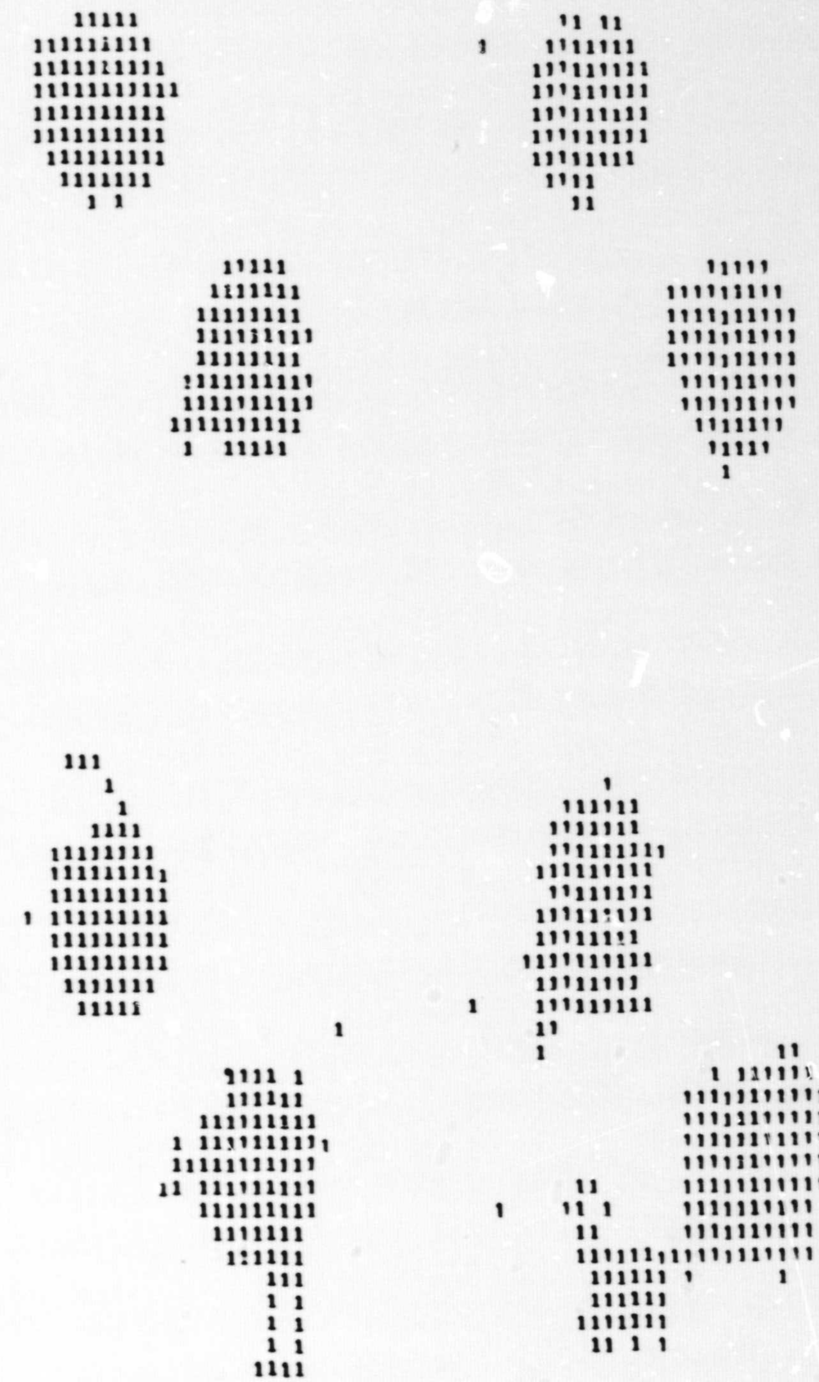


Figure 8



Let $c_{ij} = 0$ if (i,j) is in some $D_{hk}^{(r)}$ in which the number of 1's is $\le \lambda n_r$; $c_{ij} = 1$ otherwise

$d_{ij} = 1$ if (i,j) is in some $D_{hk}^{(r)}$ in which the number of 1's is $\ge (1-\lambda)n_r$; $d_{ij} = 0$ otherwise

and let $G_{r,\lambda}$ be the operation which takes (a_{ij}) into (e_{ij}) defined by

$e_{ij} = 0$ if $c_{ij} = d_{ij} = 0$
$e_{ij} = 1$ if $c_{ij} = d_{ij} = 1$
$e_{ij} = a_{ij}$ otherwise.

In other words, $e_{ij} = 0$ if (i,j) is in some neighborhood consisting mostly of 0's, and in no neighborhood consisting mostly of 1's; $e_{ij} = 1$ if the opposite is true; and $e_{ij} = a_{ij}$ if (i,j) is in neither type or in both types of neighborhood. Evidently, this operation can, in principle, eliminate noise even if it is close to the edge of the signal. For example, it can turn a "1" into a "0" if <u>some</u> neighborhood containing it consists mostly of 0's, even though the neighborhood centered at it does not.

Figures 9-15 show the results of applying $G_{r,\lambda}$ to Figure 1, for the same values of r and λ as in Figures 2-8. It will be noted that the noise cleaning power of $G_{r,\lambda}$ is generally higher than that of the corresponding $F_{r,\lambda}$. (Even the weakest G's treat streaks as noise; note, however, that for r = 1, diagonal streaks are treated more harshly, since a 3-by-3 square can intersect such a streak in only one element, while it must intersect a horizontal

Figure 9

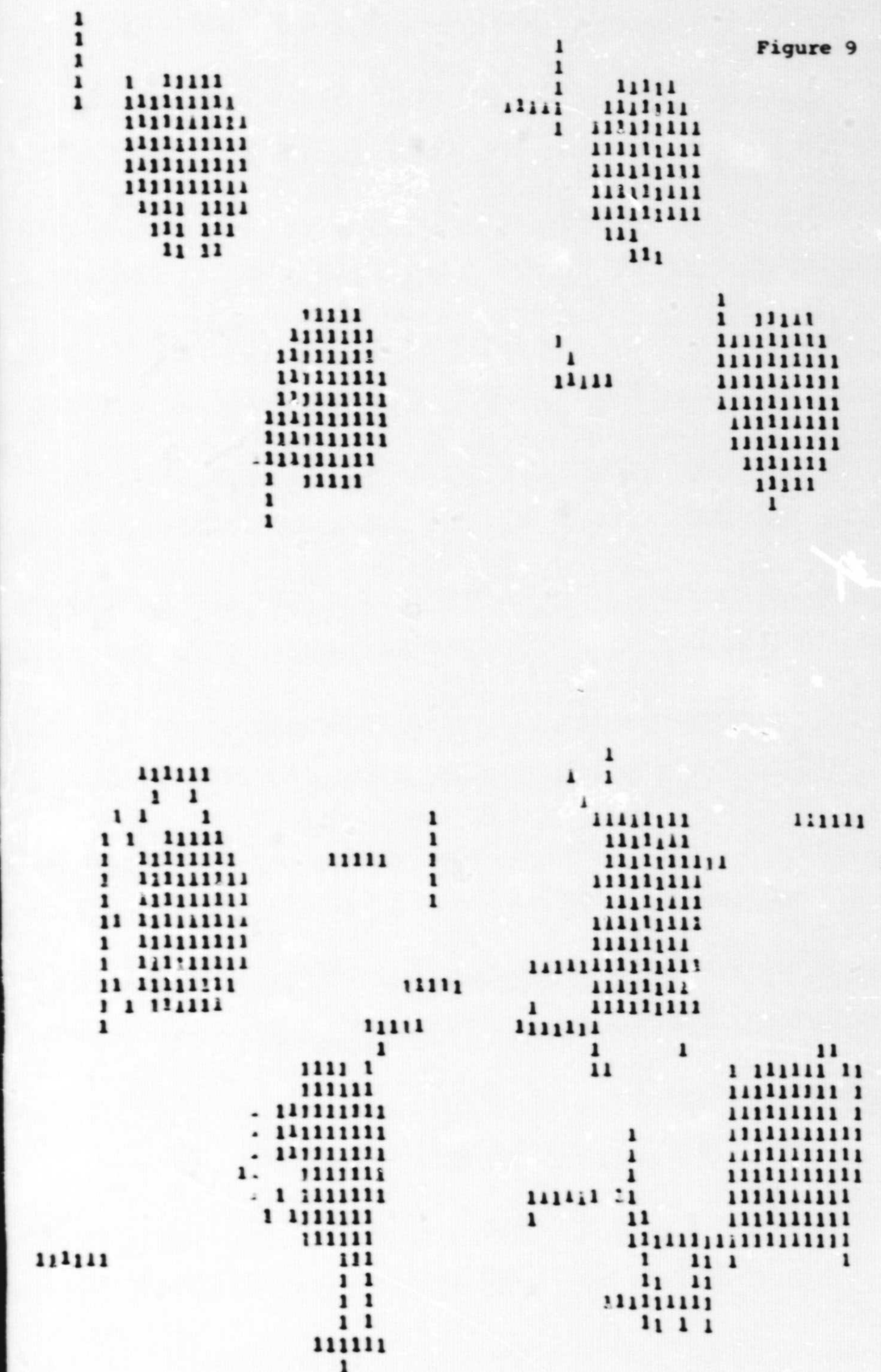


Figure 10

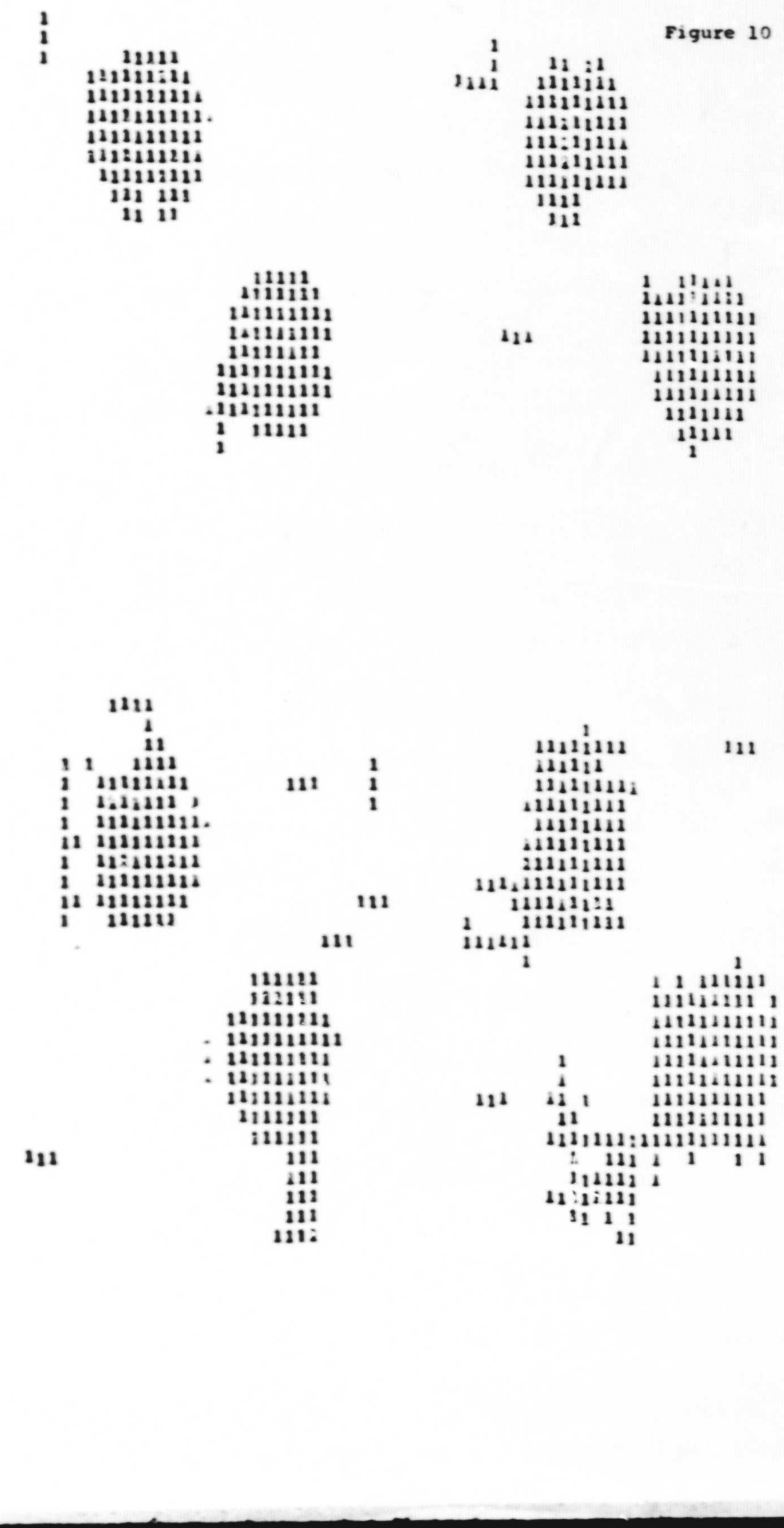
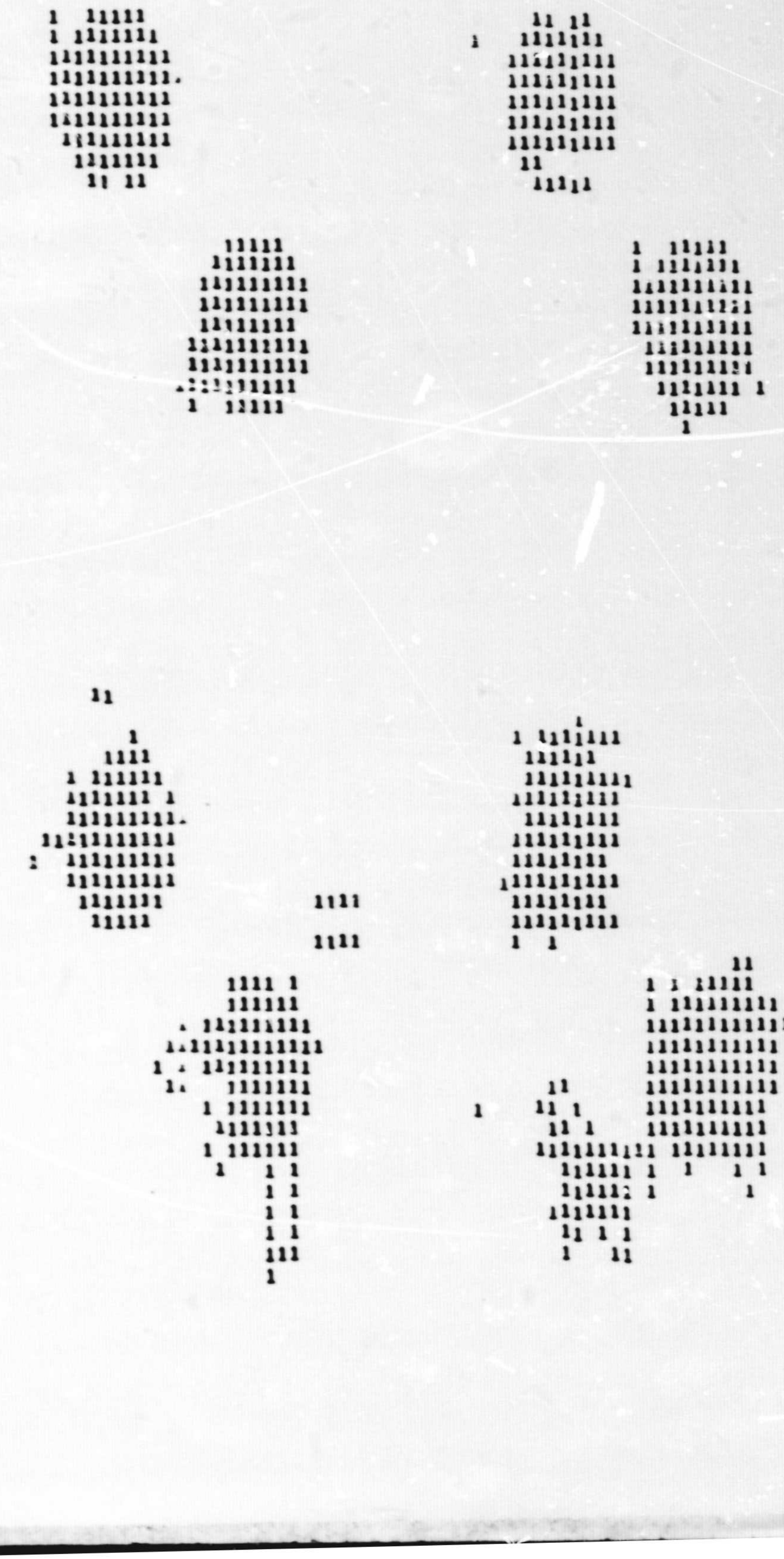


Figure 11



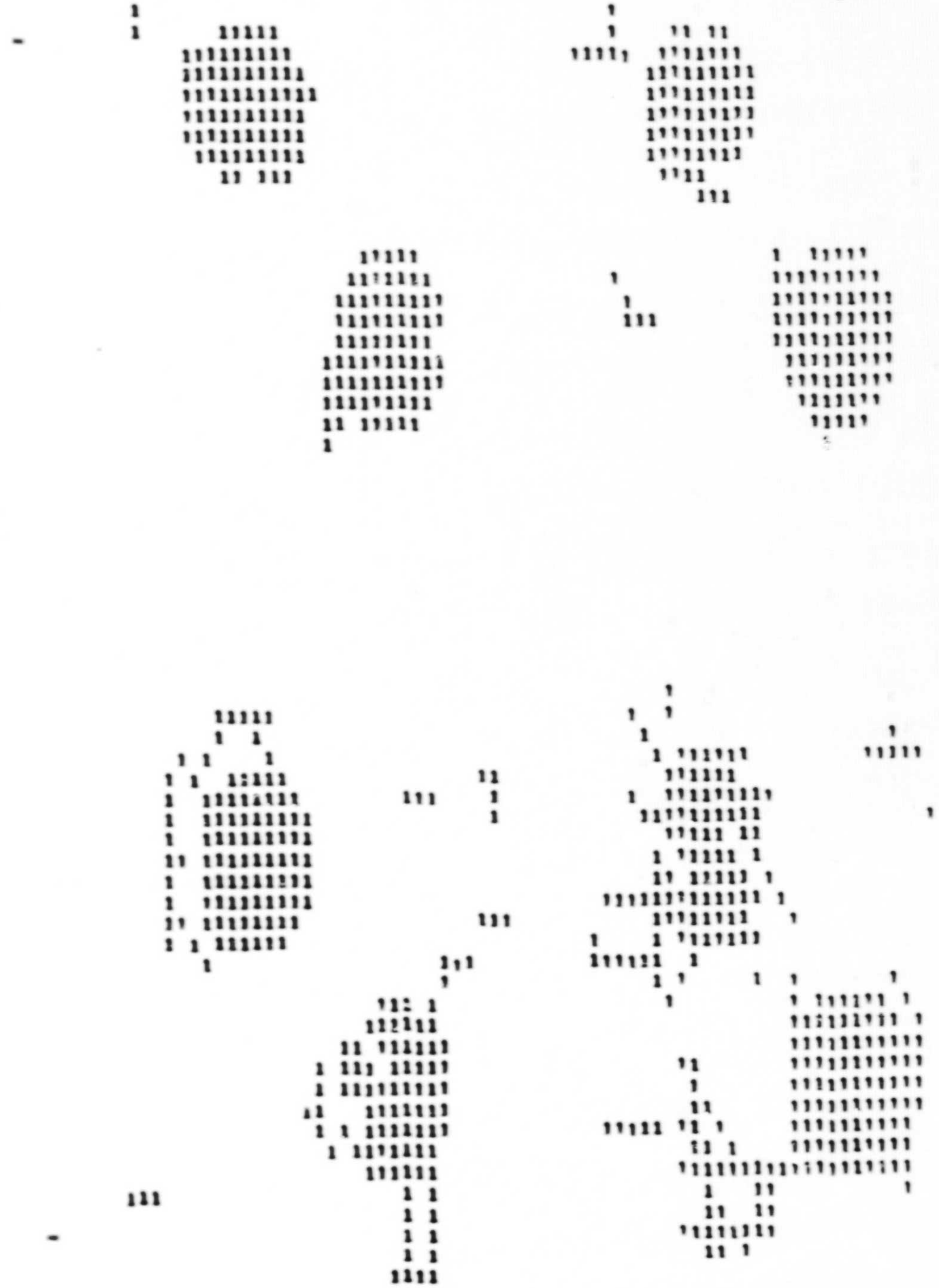


Figure 12

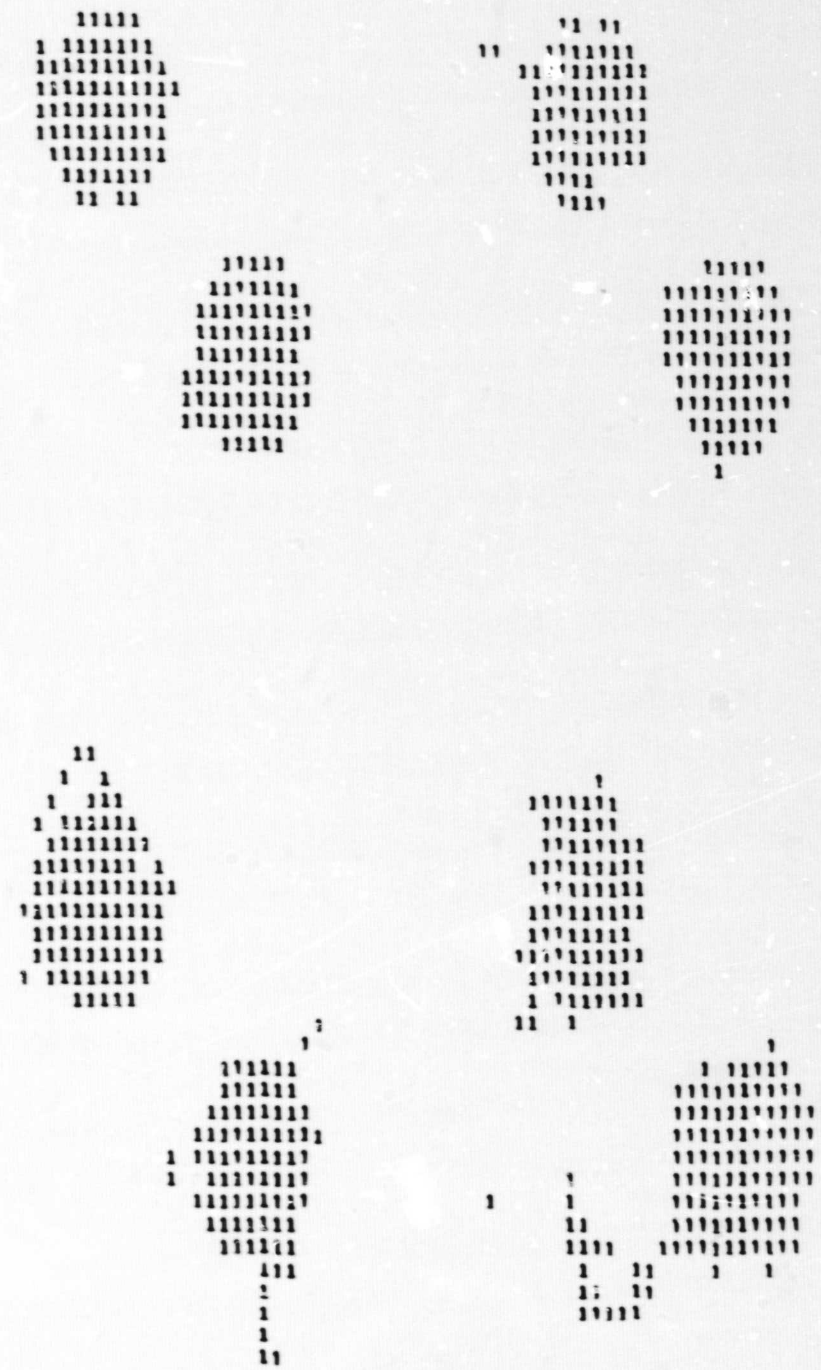


Figure 13

Figure 14

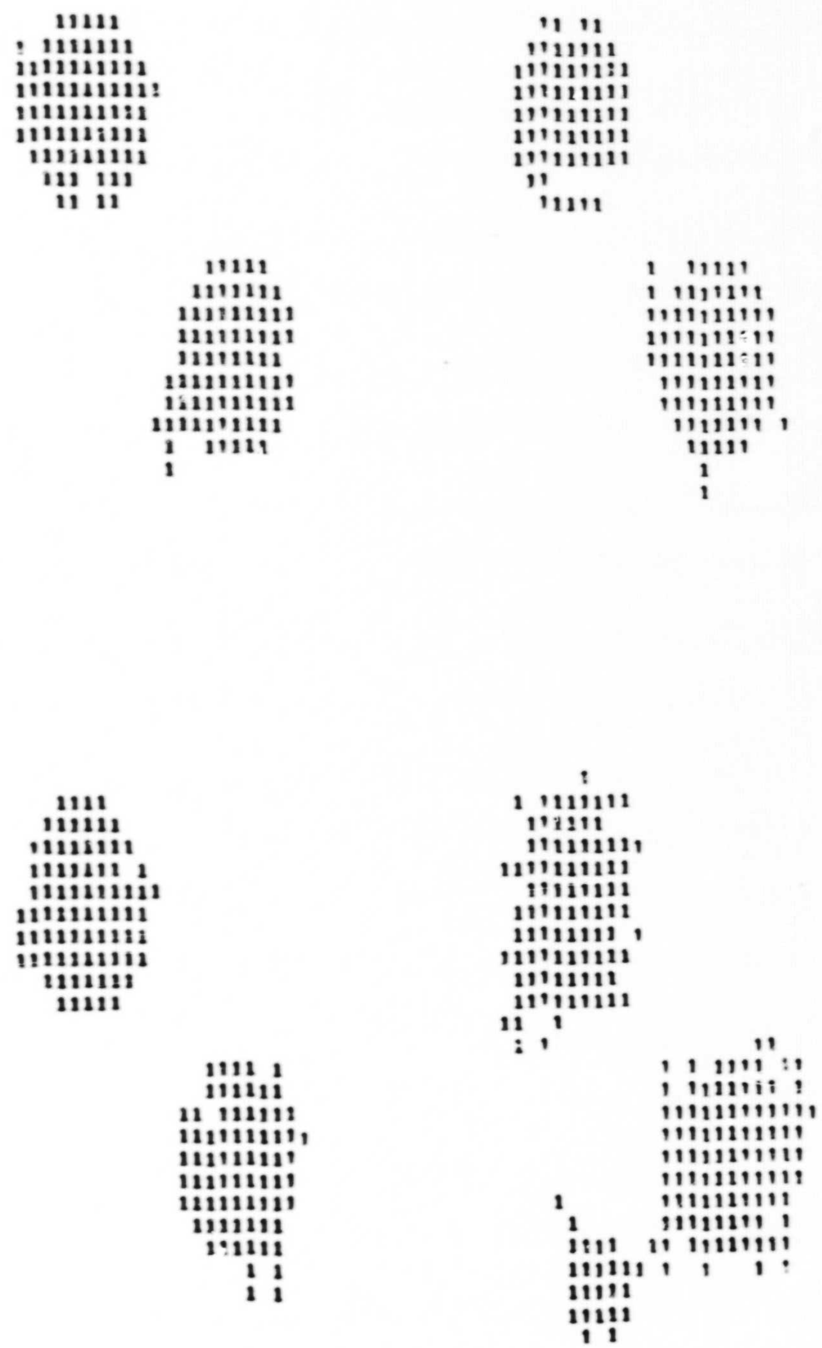
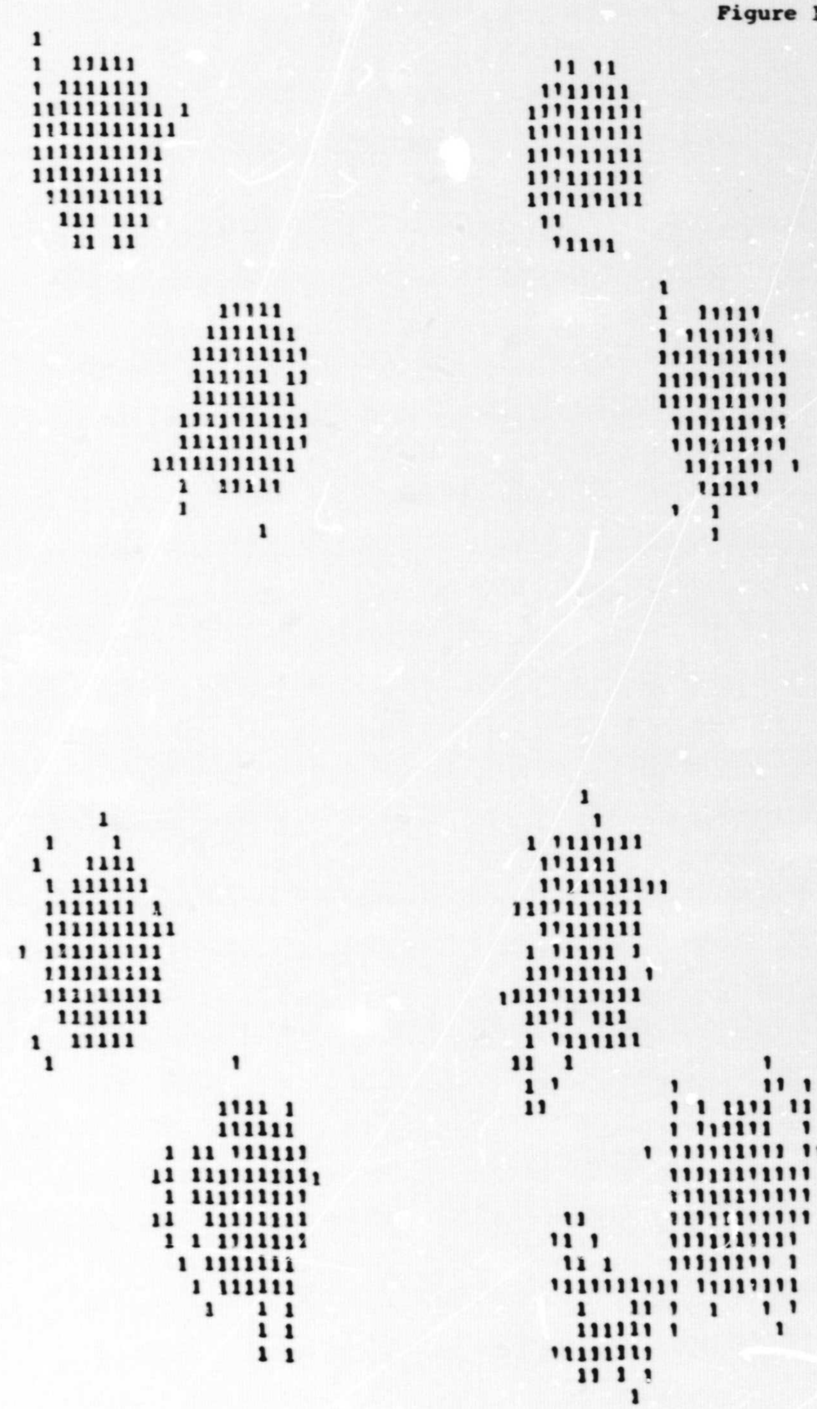


Figure 15



or vertical streak in at least three elements.) Note also that for $r = 2$ and $\lambda = 0.4$, G is actually poorer than F, since for large r and λ it becomes increasingly easy to have $c_{ij} = 0$ and $d_{ij} = 1$ at the same point, so that $e_{ij} = a_{ij}$ and noise is not cleaned.

We can also consider another approach to noise cleaning in which "propagation" processes are used. Specifically, let H be the operation which changes 0's to 1's if any of their eight neighbors are 1's, and let $\bar{H}$ similarly change 1's to 0's if any of their eight neighbors are 0's. Let $(f_{ij}) = H(\bar{H}^{(r+1)}(H^{(r)}(a_{ij})))$, $(g_{ij}) = \bar{H}(H^{(r+1)}(\bar{H}^{(r)}(a_{ij})))$, and let H_r be the operation which takes a_{ij} into (h_{ij}) defined by

$h_{ij} = 0$ if $f_{ij} = g_{ij} = 0$

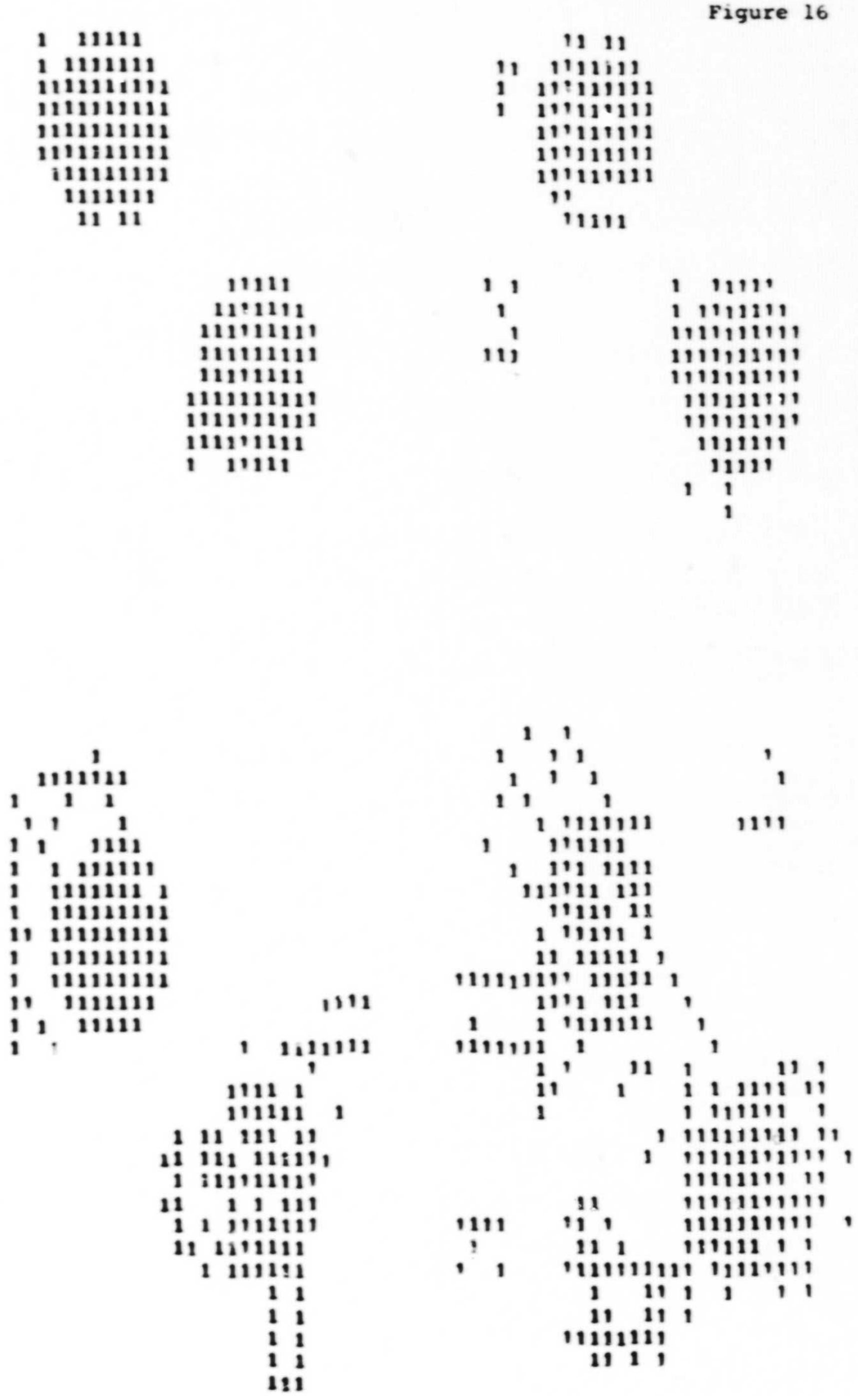
$h_{ij} = 1$ if $f_{ij} = g_{ij} = 1$

$h_{ij} = a_{ij}$ otherwise.

In other words, we obtain (f_{ij}) from (a_{ij}) by "expanding" (or "propagating") the 1's r times, then "re-contracting" them (or equivalently, expanding the 0's) r + 1 times, and finally re-expanding the 1's once. Evidently, an isolated "1" first expands under this treatment into a square of radius r, but then contracts and vanishes completely. However, if the "1" belongs to a cluster of 1's which "fuses" (see [8]) under the expansion, it will not vanish under re-contraction. Similarly, we obtain (g_{ij}) from (a_{ij}) by first expanding the 0's (or equivalently, contracting the 1's) r times, then contracting them r + 1 times, and finally re-expanding them once. Clearly an isolated "1" will vanish

immediately under this treatment, so that for such a "1" we have $h_{ij} = f_{ij} = g_{ij} = 0$, and analogously for isolated 0's; but in a region where 1's and 0's are mixed together, f_{ij} and g_{ij} will usually differ, so that we will have $h_{ij} = a_{ij}$. Note that in general, if there are many 1's in a neighborhood of radius 2r of a given "1", expansion of the 1's by r will cause "fusion" to take place, so that we will have $f_{ij} = 1$ and the given "1" will not be changed to a "0" by H_r. Figure 16 shows the result of applying H_1 to Figure 1.

Figure 16



3. Elongation analysis

The "propagation" scheme just described, like the neighbor counting schemes, treats both dots and streaks as noise. In this section, a more complex scheme is described, based on propagation, which treats the dots (and eventually, even the large circles) as noise, and the streaks as signal.

To see the basic idea underlying this scheme [8], consider the effect of contracting an object (e.g., the set of 1's in a digital picture) and then re-expanding it by the same amount. It is easily seen that the re-expanded object cannot contain any elements which were not in the original object. On the other hand, it need not contain every element of the original; for example, if an object is contracted by an amount greater than its radius, it disappears completely, so that there is nothing left to re-expand.

Suppose that the object S is contracted by k steps (i.e., the operation which turns 1's into 0's if they have 0's as neighbors is applied k times to the picture) and then re-expanded by k steps (0's having 1's as neighbors are turned into 1's, k times). Let S_k be the result of this contraction and re-expansion (so that S_k is a subset of S), and let $\bar{S}_k = S - S_k$ be the set of elements of S which are <u>not</u> in S_k. Let $\bar{A}_k$ be a connected component of $\bar{S}_k$. Since every element of the subset $\bar{A}_k$ of S disappeared under contraction and re-expansion by k steps, $\bar{A}_k$ cannot be more than 2k "wide". Suppose that the area of $\bar{A}_k$ is (say) $10k^2$ or greater. Since its "width" is at most 2k, its "length" must thus be at least 5k, i.e., 2½ times its "width", making it elongated. Here "length" and "width" can be taken literally if $\bar{A}_k$ is a rectangle; but in general too, it is reasonable to call a connected set elongated if its area is more than 2½ times the square of its greatest "width" (where the "width" of a set at a point might be defined as twice the distance from the point to the set's border).

The criterion just described requires refinement before it can be used as reasonable definition of "elongated". Note, in fact, that the object

1111

disappears under contraction by one step, but since its area is only 4, it would not be called elongated. This is because the criterion was designed on the basis of objects which are elongated in spite of being as wide as possible. Indeed, the object

11
11

also disappears under contraction by one step, and has area 4, but is certainly not elongated. If an object disappears under one step of contraction, the criterion calls it elongated only if its area is at least 10, since it may be 2 wide, but this criterion is too severe for objects which are only 1 wide. In general, the criterion must be too severe for objects of width 2k-1, if it is not to be too lenient for objects of width 2k; but as k increases, this difficulty diminishes.

The objection just raised can be overcome if the definition of the contraction process can be modified so that objects of width 2k-1 and 2k require different numbers of contraction steps before they disappear. In particular, suppose that the process is redefined as follows: on odd-numbered steps, change 1's to 0's if they have 6 or more 0's as neighbors; on even-numbered steps, change 1's to 0's if they have any 0's as neighbors. It is easily seen that a rectangle n elements wide takes n steps to disappear under this modified process (rather than $\frac{n}{2}$ or $\frac{n+1}{2}$, depending on whether n is even or odd, as under the process described originally). If an object is contracted by k steps using the new procedure, and then re-expanded(by changing 0's to 1's if they have any 1's as neighbors, $\lfloor \frac{k}{2} \rfloor$ times, where $\lfloor \frac{k}{2} \rfloor$ is the largest integer not greater than $\frac{k}{2}$), one can again consider connected components of what fails to reappear, and define a component to be elongated if its area is(say) $2\frac{1}{2}k^2$ or greater.

A further objection to this class of elongatedness criteria is their sensitivity to noise. Suppose, for example, that a (2k+1)-by-(2k+1) square of 1's contains a single "0" at its center; then it disappears under contraction by only k+1 steps (rather than 2k+1 steps), so that (for k ≥ 2) it would be called elongated by the criterion just described, which (at least for large k) seems unreasonable. On the other hand, if the "hole" in the center of the square is large with respect to k, it is not unreasonable to call the "square" elongated, since it is really an annulus.

This last objection can be overcome by carrying out a more elaborate propagation procedure, in which elongatedness criteria are applied, alternatingly, to both the 1's and the 0's in the given picture P_0, and connected components which are not found to be elongated are treated as noise. Specifically, the 1's in P_0 are contracted by one step as described earlier*, and the small (=non-elongated) connected components which disappear (=fail to reappear) remain as 0's, while <u>the elongated components are changed back to 1's.</u> Conversely, the 0's in P_0 are contracted by one step, and their non-elongated components which disappear become 1's, while their elongated components are changed back to 0's. Let the resulting pictures be P_1 and P_1', respectively. At the second stage, the 1's in P_1' are contracted by <u>two</u> steps and re-expanded, and their non-elongated components which fail to reappear remain as 0's, while the elongated components are changed back to 1's; and analogously for the 0's in P_1. Let the results of this stage be P_2 and P_2', respectively, and repeat the process, contracting the 1's in P_2 (the 0's in P_2') by <u>three</u> steps and re-expanding; and so on. In the example of the square with a hole at its center considered in the preceding paragraph, it is easily seen that if the width of the hole is less than 1/3 that of the square, the hole will disappear under shrinking and re-expansion of 0's before the square (annulus) disappears under shrinking and re-expansion of 1's; so that

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* Note that for $k = 1$ we have $\left[\frac{k}{2}\right]=0$, so that at this step there is actually no re-expansion.

if the hole itself is not elongated, the square will not be called elongated. More generally, the presence of holes will not make an object elongated unless they are at least as wide as the pieces of object which separate them.

In practice, it turns out that the threshold used to decide whether a component is elongated should not be the same as that used to decide whether a component is "noise". In the examples which follow, a component is allowed to disappear only if its area is $\leq k^2$ (rather than $2\frac{1}{4}\, k^2$ as would be required for an elongatedness decision).

Figures 17-18 show the results of one-step contraction and re-expansion of both the 1's and the 0's in Figure 1: (a) Connected components of what failed to reappear, successively labelled 1, 2,...,9, A, B,...,V, 1, 2,..., etc. (b) Table of the areas of the components (c) Result of deleting "noise" components which failed to reappear. Figures 19-20 show the next stage (two step contraction and re-expansion). If the process were repeated further, little would change until the 9th stage, when the circles themselves would disappear. The results of applying the process to a cloud cover picture, for 15 stages, are shown in Figure 21: (a) Original picture (cloud elements denoted by blanks, noncloud by 1's); (b) Noncloud elements labelled 1, 2,... according to the stage at which they fail to reappear and are deleted ("X" denotes elements which have not been deleted even at the 15th stage); (c) Noncloud

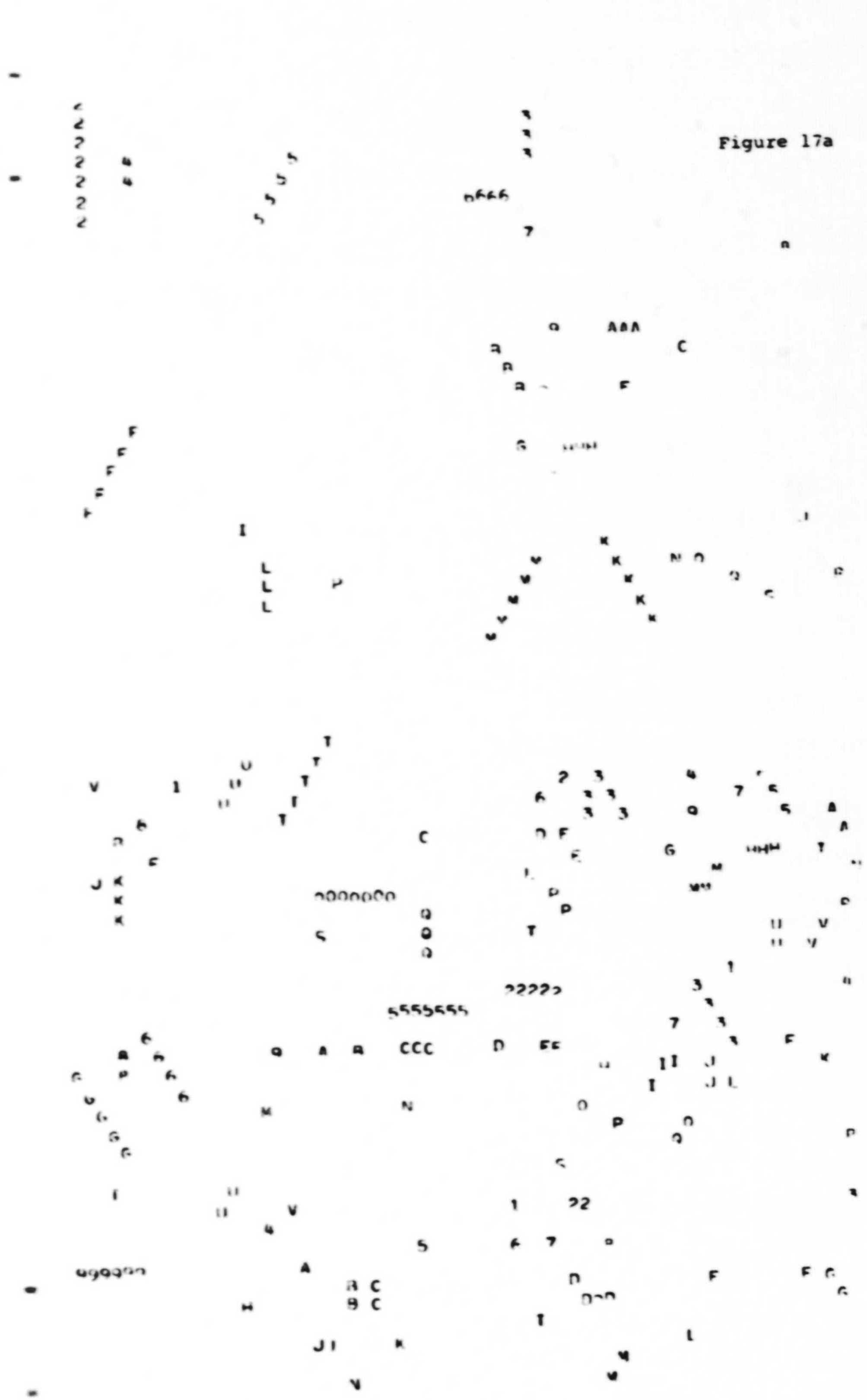


Figure 17a

NUMBER	SYMBOL	AREA	RATIO
2	2	7	7.000
3	3	3	3.000
4	4	2	2.000
5	5	4	4.000
6	6	4	4.000
7	7	1	1.000
8	8	1	1.000
9	9	1	1.000
10	A	3	3.000
11	B	3	3.000
12	C	1	1.000
13	D	1	1.000
14	E	1	1.000
15	F	5	5.000
16	G	1	1.000
17	H	3	3.000
18	I	1	1.000
19	J	1	1.000
20	K	5	5.000
21	L	3	3.000
22	M	5	5.000
23	N	1	1.000
24	O	1	1.000
25	P	1	1.000
26	Q	1	1.000
27	R	1	1.000
28	S	1	1.000
29	T	5	5.000
30	U	3	3.000
31	V	1	1.000
32	1	1	1.000
33	2	1	1.000
34	3	5	5.000
35	4	1	1.000
36	5	3	3.000
37	6	1	1.000
38	7	1	1.000
39	8	1	1.000
40	9	1	1.000
41	A	2	2.000
42	B	1	1.000
43	C	1	1.000
44	D	1	1.000
45	E	2	2.000
46	F	1	1.000
47	G	1	1.000
48	H	3	3.000
49	I	1	1.000
50	J	1	1.000
51	K	3	3.000
52	L	1	1.000
53	M	3	3.000
54	N	1	1.000
55	O	7	7.000
56	P	2	2.000
57	Q	3	3.000
58	R	1	1.000

Figure 17b

59	S	1	1.000
60	T	1	1.000
61	U	2	2.000
62	V	2	2.000
63	1	1	1.000
64	2	5	5.000
65	3	4	4.000
66	4	1	1.000
67	5	7	7.000
68	6	4	4.000
69	7	1	1.000
70	8	2	2.000
71	9	1	1.000
72	A	1	1.000
73	B	1	1.000
74	C	3	3.000
75	D	1	1.000
76	E	2	2.000
77	F	1	1.000
78	G	5	5.000
79	H	1	1.000
80	I	3	3.000
81	J	2	2.000
82	K	1	1.000
83	L	1	1.000
84	M	1	1.000
85	N	1	1.000
86	O	1	1.000
87	P	1	1.000
88	Q	2	2.000
89	R	1	1.000
90	S	1	1.000
91	T	1	1.000
92	U	2	2.000
93	V	1	1.000
94	1	1	1.000
95	2	2	2.000
96	3	1	1.000
97	4	1	1.000
98	5	1	1.000
99	6	1	1.000
100	7	1	1.000
101	8	1	1.000
102	9	6	6.000
103	A	1	1.000
104	B	2	2.000
105	C	2	2.000
106	D	4	4.000
107	E	1	1.000
108	F	1	1.000
109	G	2	2.000
110	H	1	1.000
111	I	1	1.000
112	J	2	2.000
113	K	1	1.000
114	L	1	1.000
115	M	2	2.000
116	N	1	1.000

Figure 17b (continued)

Figure 17c

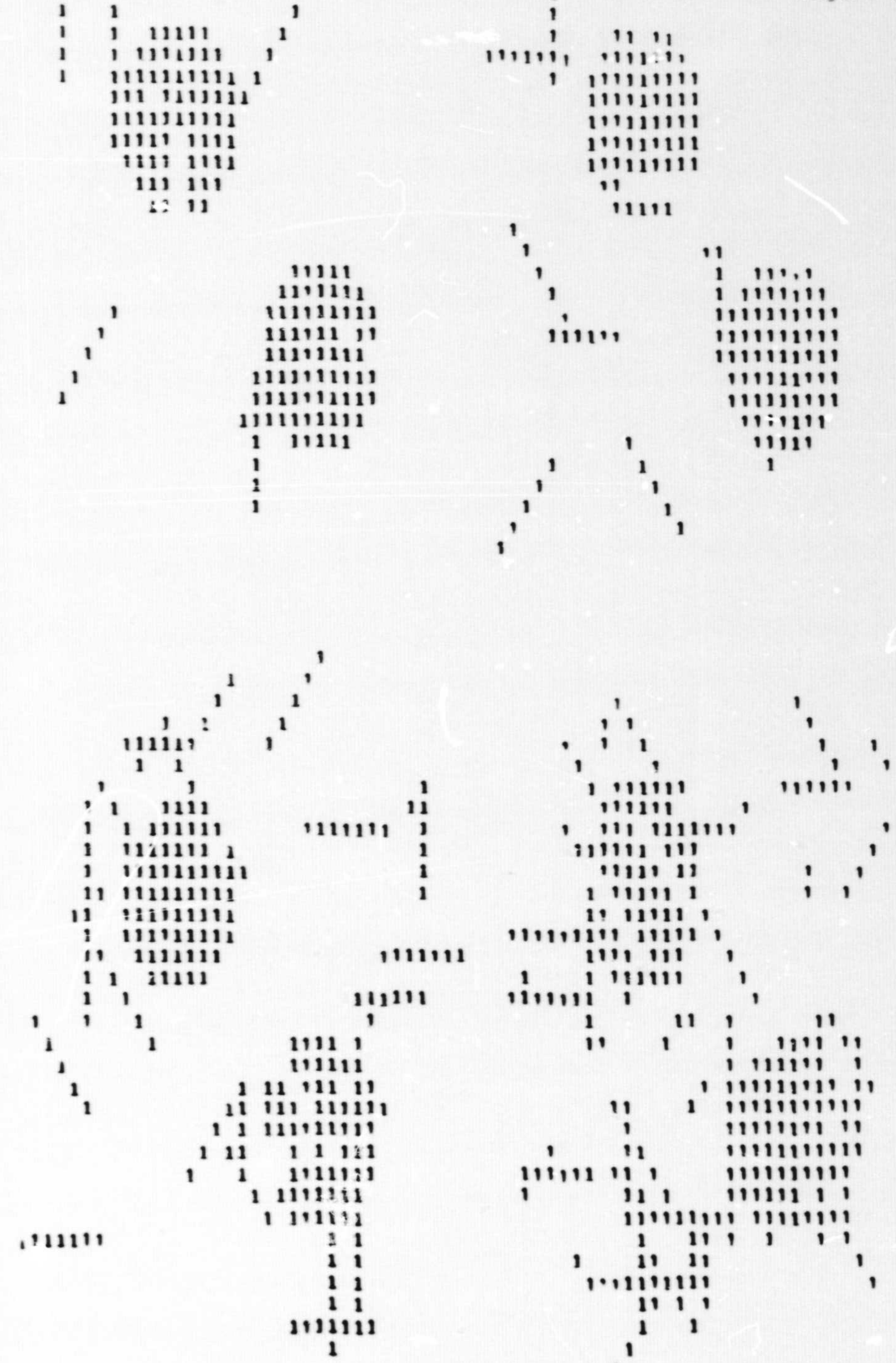


Figure 18a

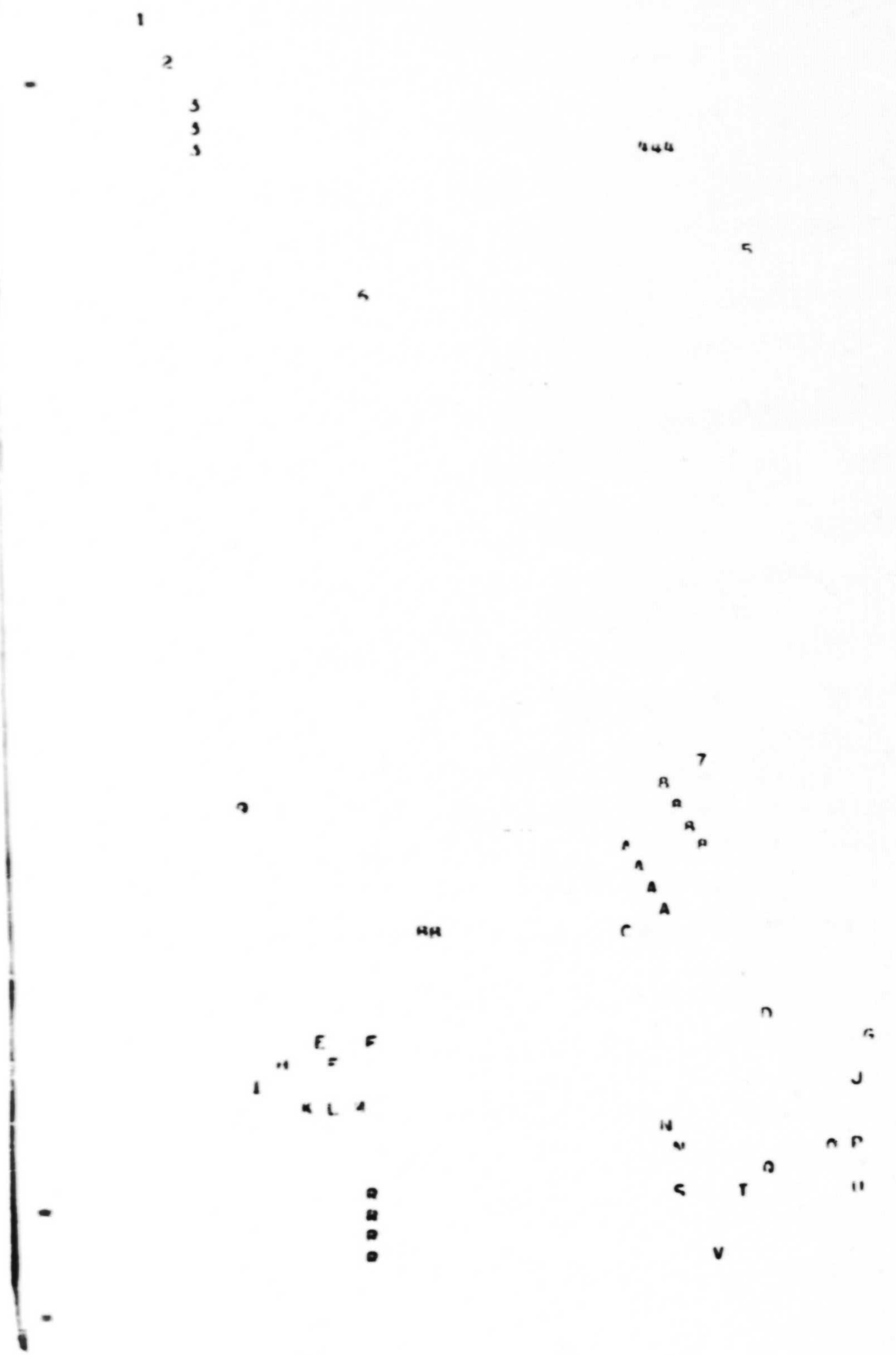


Figure 18b

NUMBER	SYMBOL	AREA	RATIO
1	1	1	1.000
2	2	1	1.000
3	3	3	3.000
4	4	3	3.000
5	5	1	1.000
6	6	1	1.000
7	7	1	1.000
8	8	4	4.000
9	9	1	1.000
10	A	4	4.000
11	B	2	2.000
12	C	1	1.000
13	D	1	1.000
14	E	2	2.000
15	F	1	1.000
16	G	1	1.000
17	H	1	1.000
18	I	1	1.000
19	J	1	1.000
20	K	1	1.000
21	L	1	1.000
22	M	1	1.000
23	N	2	2.000
24	O	1	1.000
25	P	1	1.000
26	Q	1	1.000
27	R	4	4.000
28	S	1	1.000
29	T	1	1.000
30	U	1	1.000
31	V	1	1.000

Figure 18c

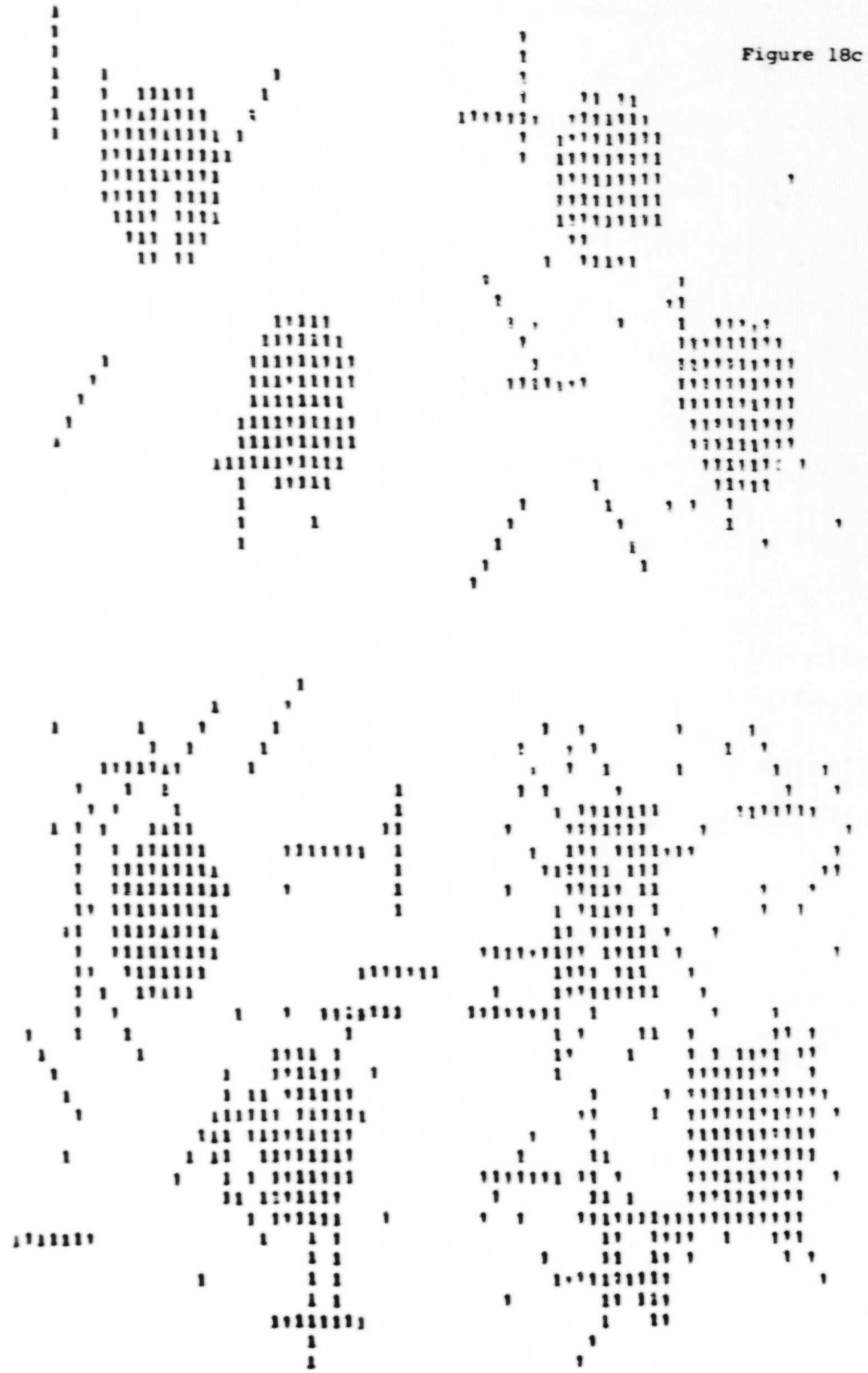
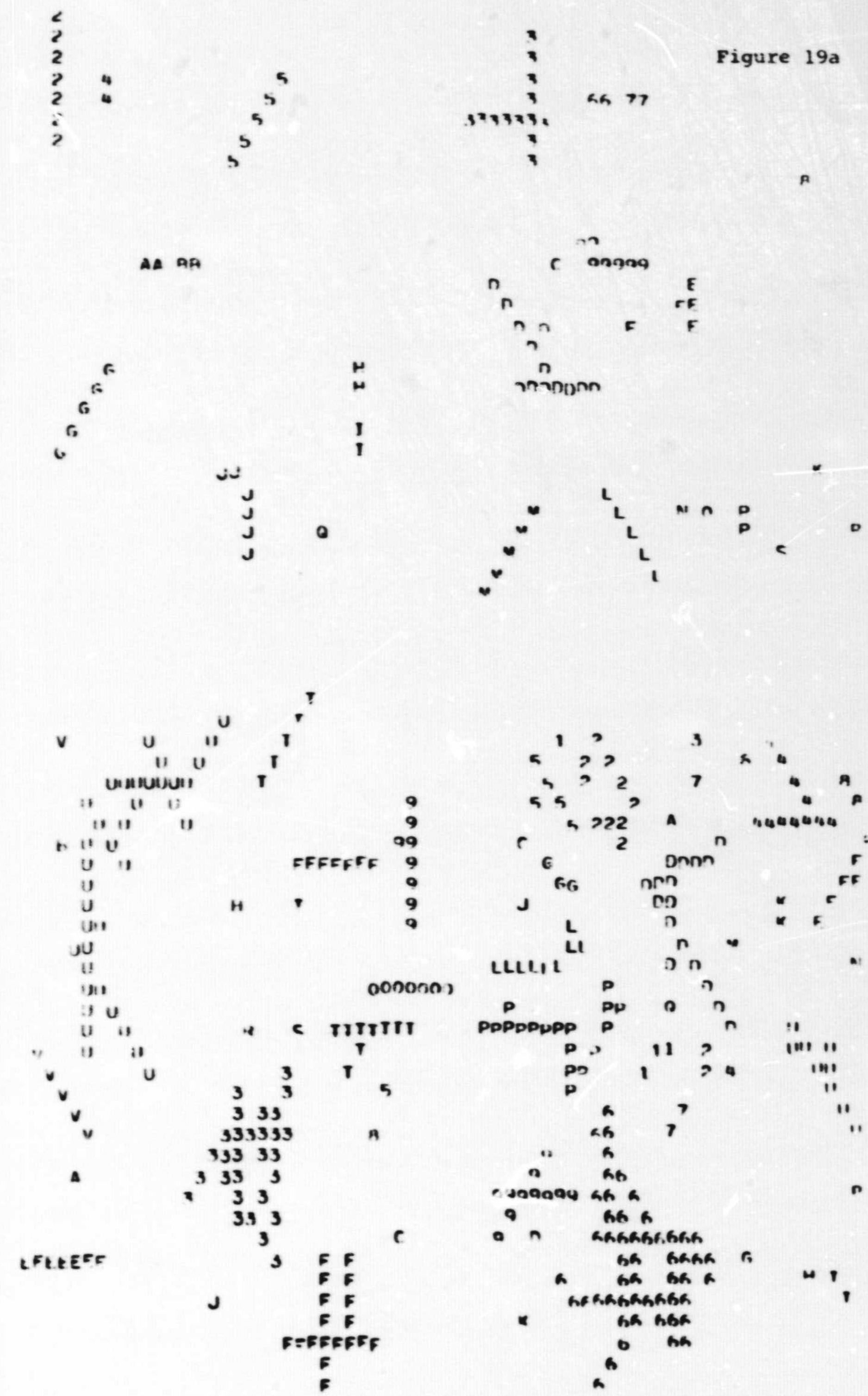


Figure 19a



NUMBER	SYMBOL	AREA	RATIO
2	2	7	1.750
3	3	13	3.250
4	4	2	.500
5	5	5	1.250
6	6	2	.500
7	7	2	.500
8	8	1	.250
9	9	7	1.750
10	A	2	.500
11	B	2	.500
12	C	1	.250
13	D	13	3.250
14	E	4	1.000
15	F	1	.250
16	G	5	1.250
17	H	2	.500
18	I	2	.500
19	J	6	1.500
20	K	1	.250
21	L	5	1.250
22	M	5	1.250
23	N	1	.250
24	O	1	.250
25	P	2	.500
26	Q	1	.250
27	R	1	.250
28	S	1	.250
29	T	5	1.250
30	U	38	9.500
31	V	1	.250
32	1	1	.250
33	2	10	2.500
34	3	1	.250
35	4	11	2.750
36	5	5	1.250
37	6	1	.250
38	7	1	.250
39	8	2	.500
40	9	8	2.000
41	A	1	.250
42	B	1	.250
43	C	1	.250
44	D	17	4.250
45	E	6	1.500
46	F	7	1.750
47	G	3	.750
48	H	1	.250
49	I	1	.250
50	J	1	.250
51	K	2	.500
52	L	9	2.250
53	M	1	.250
54	N	1	.250
55	O	7	1.750
56	P	18	4.500
57	Q	1	.250
58	R	1	.250

Figure 19b

59	S	1	.250
60	T	9	2.250
61	U	9	2.250
62	V	5	1.250
63	1	3	.750
64	2	2	.500
65	3	29	7.250
66	4	1	.250
67	5	1	.250
68	6	53	13.250
69	7	2	.500
70	8	1	.250
71	9	11	2.750
72	A	1	.250
73	B	1	.250
74	C	1	.250
75	D	1	.250
76	E	7	1.750
77	F	18	4.500
78	G	1	.250
79	H	1	.250
80	I	2	.500
81	J	1	.250
82	K	1	.250

Figure 19b (continued)

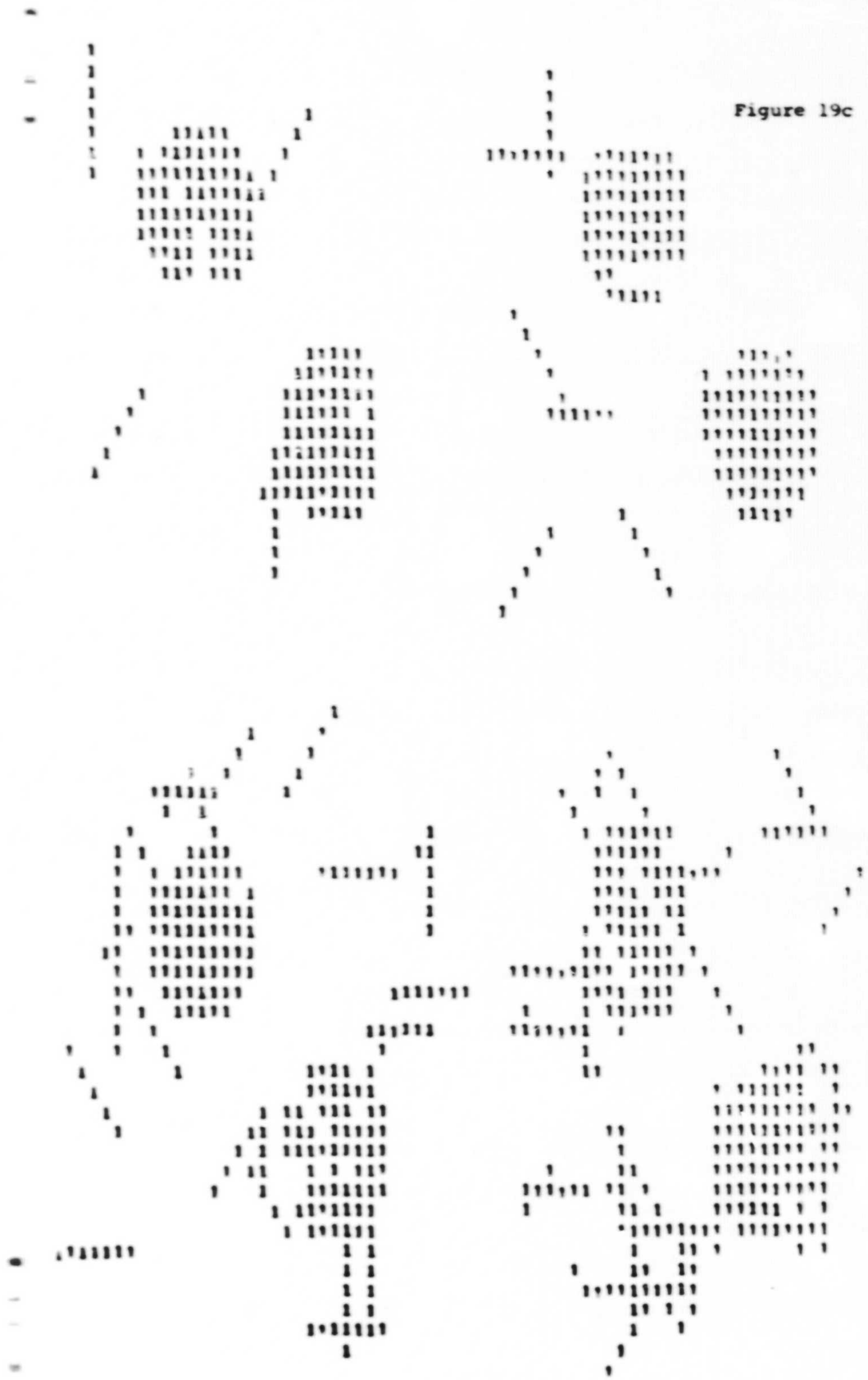


Figure 19c

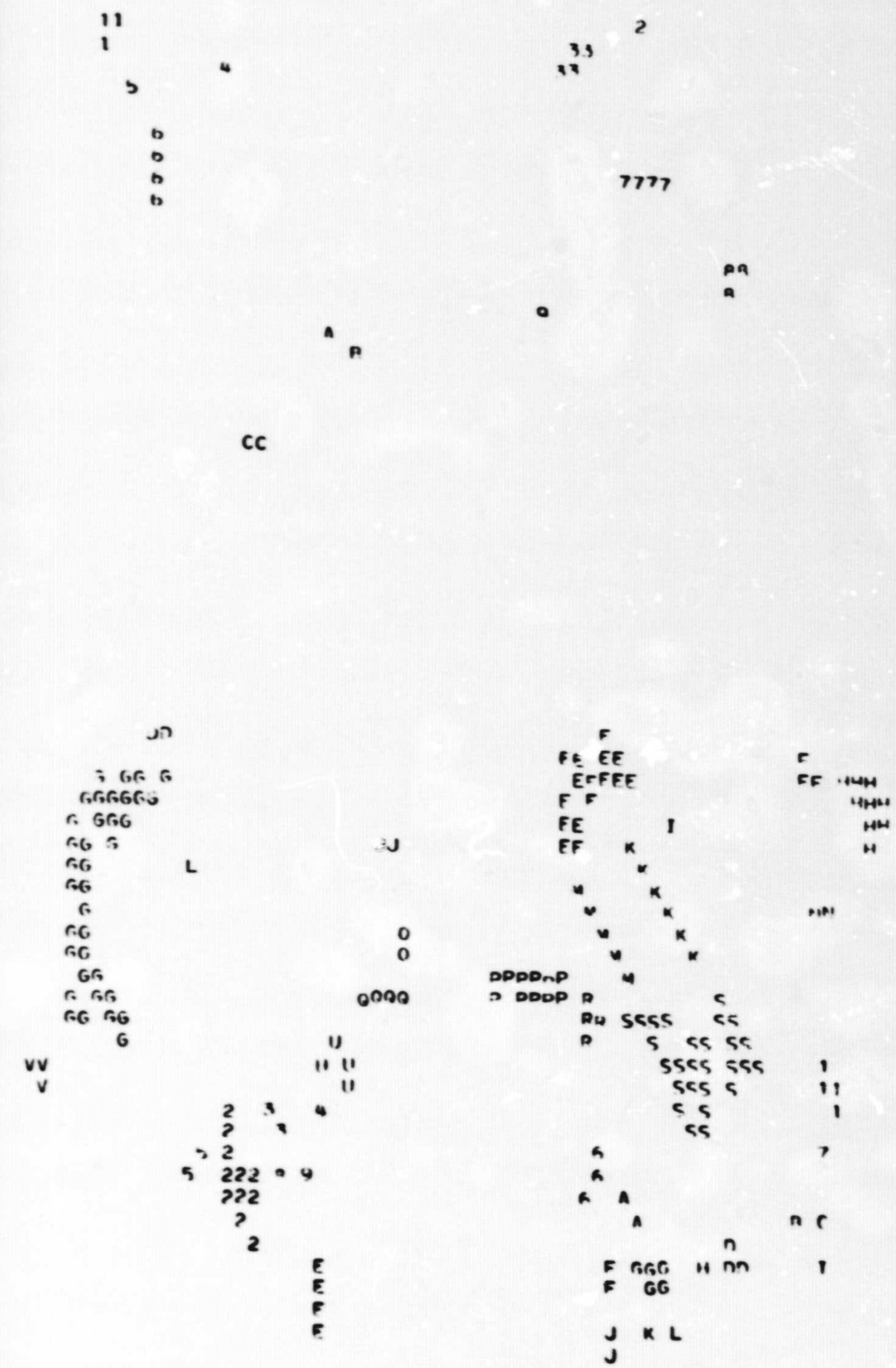


Figure 20a

NUMBER	SYMBOL	AREA	RATIO
1	1	3	.750
2	2	1	.250
3	3	4	1.000
4	4	1	.250
5	5	1	.250
6	6	4	1.000
7	7	4	1.000
8	8	3	.750
9	9	1	.250
10	A	1	.250
11	B	1	.250
12	C	2	.500
13	D	2	.500
14	E	16	4.000
15	F	3	.750
16	G	36	9.000
17	H	9	2.250
18	I	1	.250
19	J	2	.500
20	K	6	1.500
21	L	1	.250
22	M	5	1.250
23	N	2	.500
24	O	2	.500
25	P	11	2.750
26	Q	4	1.000
27	R	4	1.000
28	S	27	6.750
29	T	1	.250
30	U	4	1.000
31	V	3	.750
32	1	4	1.000
33	2	11	2.750
34	3	2	.500
35	4	1	.250
36	5	2	.500
37	6	3	.750
38	7	1	.250
39	8	1	.250
40	9	1	.250
41	A	2	.500
42	B	1	.250
43	C	1	.250
44	D	3	.750
45	E	4	1.000
46	F	2	.500
47	G	5	1.250
48	H	1	.250
49	I	1	.250
50	J	2	.500
51	K	1	.250
52	L	1	.250

Figure 20b

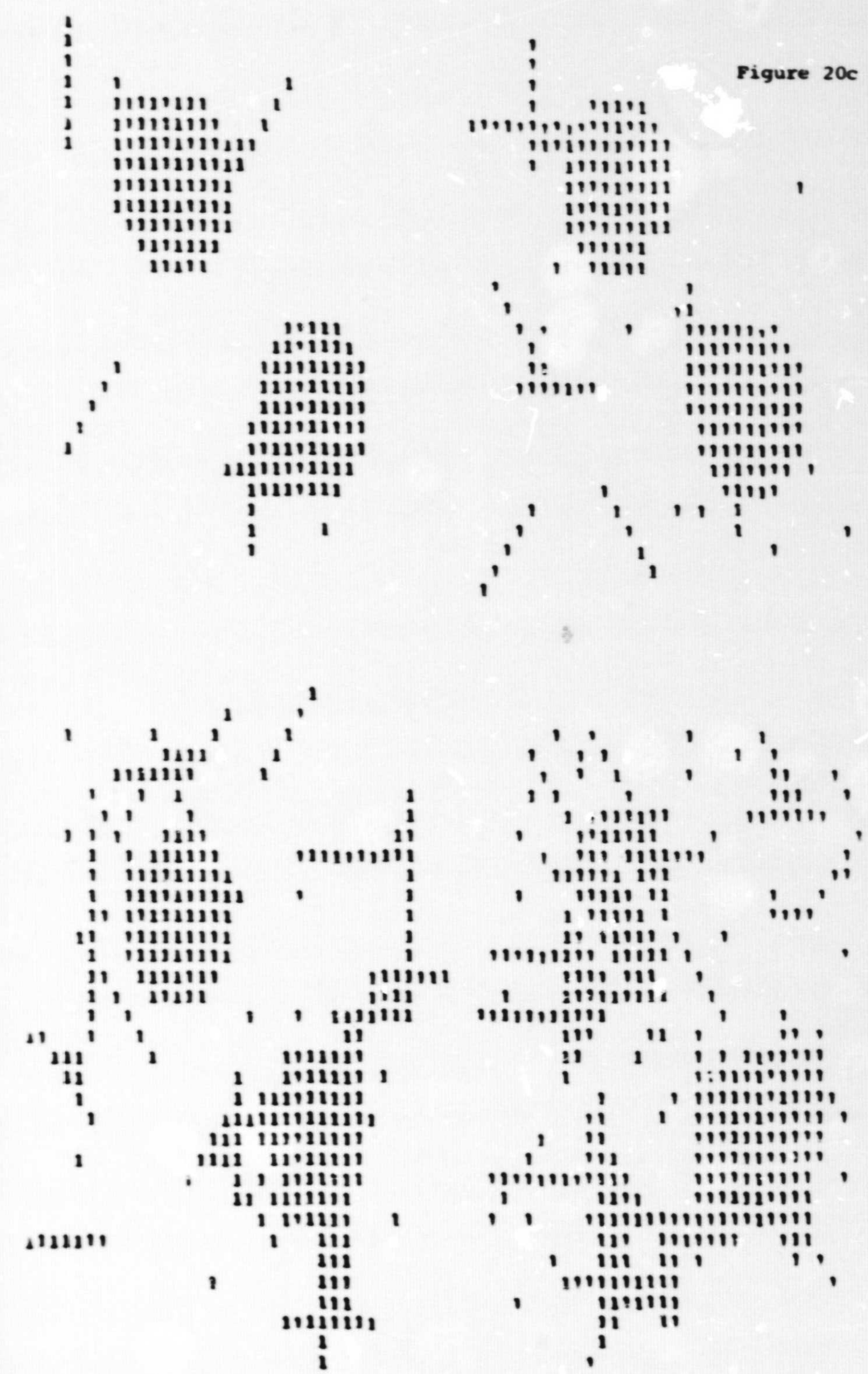


Figure 20c

elements labelled 1, 2,... according to the stage at which they fail to reappear and are found to belong to elongated components; (d-e) analogously for cloud elements. [Note that this noise cleaning method may occasionally make a "cut" across a region at a point of constriction; this is probably aggravated by the fact that the expansion and contraction operations being used are "non-Euclidean" (a "circle" of radius k is actually a square), so that the constrictions look narrower to the operations than they actually are.]

Figure 21a

Figure 21b

Figure 21c

Figure 21d

Figure 21e

4. Isolatedness

A propagation scheme can also be used to define isolated elements in a binary-valued digital picture [8]. Suppose, in fact, that the 1's in such a picture are "expanded" (in the simple sense of Section 2) by k steps, and let B_k be a connected component of 1's in the resulting picture. If B_k arises from a single "1" in the original picture which was at distance greater than 2k from the nearest other "1", it will be a square of side 2k + 1 (assuming that in the expansion, an element is regarded as having eight neighbors), and so will have area $(2k+1)^2$. Conversely, if a component has area close to $(2k+1)^2$, it must have arisen from a small, closely clustered set of 1's in the original picture which had no other 1's within distance 2k of them -- in short, such a component must have arisen from an isolated set of 1's. Under some circumstances, one might want to preserve small objects in a picture when they are isolated, but not otherwise; the procedure just outlined can be used for this purpose. Figure 22 shows, and tabulates the areas of, the connected components resulting when the 1's in Figure 1 are expanded by 1 and by 2 steps. Note that an elongated object will never be called elongated by this scheme (e.g., components 2, 7, 8, A in Figure 22a, and 4 in Figure 22b).

Figure 22a

Figure 22b

NUMBER	SYMBOL	AREA	RATIO
1	1	9	1.000
2	2	27	3.000
3	3	415	46.111
4	4	145	16.111
5	5	9	1.000
6	6	139	15.444
7	7	29	3.222
8	8	29	3.222
9	9	1439	159.889
10	A	24	2.667
11	B	9	1.000

Figure 22c

NUMBER	SYMBOL	AREA	RATIO
1	1	639	25.560
2	2	523	20.920
3	3	25	1.000
4	4	61	2.440
5	5	2014	80.560

Figure 22d

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